

2 Mechanics

Date

2.1 Motion

Distance is a scalar.

Displacement is a vector.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{ms}^{-1}$$

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\text{ms}^{-1}$$

Instantaneous and Average Values

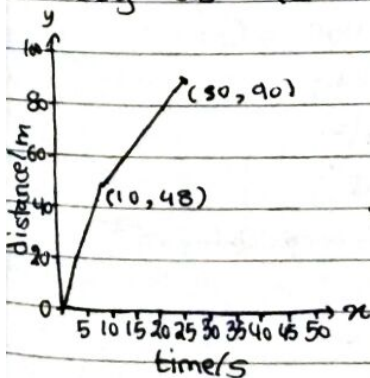
Speedometer gives instantaneous speed.

Instantaneous speed - Rate of change of position with respect to time.

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} \quad \frac{ds}{dt} \text{ or } \frac{\Delta d}{\Delta t}$$

Examples

The graph shows distance and time of a boy as he runs.



b) $\frac{90}{30} = 3 \text{ ms}^{-1}$
 $= 3.0 \text{ ms}^{-1}$

a) instantaneous speed

i) 5.0s

$$= \frac{48}{10} = 4.8 \text{ ms}^{-1}$$

ii) 20s

$$= \frac{42}{20} = 2.1 \text{ ms}^{-1}$$

Examples

Japanese N700 train has acceleration of 0.72 ms^{-2} . How many seconds will it take the N700 to reach its maximum speed of 300 km h^{-1} on the Sanyo Shinkansen route?

$$0.72 = \frac{300 - 0}{t}$$

$$0.72t = 300$$

$$t = \frac{300 \times 1000}{60 \times 60} = \frac{300000}{3600} = 83.3 \text{ ms}^{-1}$$

$$= 83 \text{ ms}^{-1}$$

$$t = \frac{83 - 0}{0.72}$$

$$t = 83 / 0.72$$

$$= 116 \text{ seconds}$$

Examples

1) A cyclist travels 16 km in 70 minutes. Calculate, in ms^{-1} , the speed.

$$16 \times 1000 = 16000 \text{ m}$$

$$70 \times 60 = 4200$$

$$\frac{16000}{4200} = 3.8 \text{ ms}^{-1}$$

2) The speed of light in a vacuum is $3.0 \times 10^8 \text{ ms}^{-1}$.

A star is 22 light years from Earth. Calculate the distance of the star from earth in km.

$$\frac{(3.0 \times 10^8) \times 60 \times 60 \times 24 \times 365}{1000}$$

$$= \frac{9.5 \times 10^{15}}{1000}$$

$$= 9.5 \times 10^{12} \text{ km in a year}$$

$$= 9.5 \times 10^{12} \times 22 = 2.1 \times 10^{14} \text{ km}$$

Acceleration ms^{-2}

acceleration = $\frac{\text{change in velocity}}{\text{time taken for change}}$

The "suvat" equations of motion

Examples

Symbol	quantity
s	displacement/distance
u	Initial velocity/speed
v	Final velocity/speed
a	Acceleration
t	time (s)

$$a = \frac{v-u}{t}$$

$$at = v-u$$

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$a = \frac{v-u}{t}$$

$$t = \frac{v-u}{a}$$

$$s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

$$= \frac{uv-u^2}{a} + \frac{1}{2}a\left(\frac{v^2+u^2-2uv}{a^2}\right)$$

$$s = \frac{uv-u^2}{a} + \frac{v^2+u^2-2uv}{2a}$$

$$2s = \frac{2uv-2u^2}{2a} + \frac{v^2+u^2-2uv}{2a}$$

$$s = \frac{2uv-2u^2 + v^2+u^2-2uv}{2a}$$

$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as$$

$$s = \left(\frac{v+u}{2}\right)t$$

Acceleration is constant!

If not, don't use these equations

1) A driver of a car travelling at 25 m s^{-1} along a road applies the brakes. The car comes to a stop in 150 m with a uniform deceleration. Calculate

- the time the car takes to stop
- the deceleration of the car

$$a) \quad s = ut + \frac{1}{2}at^2$$

$$s = 25t + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$0 = 625 + 2(a)(150)$$

$$0 = 625 + 300a$$

$$-625 = 300a$$

$$\frac{-625}{300} = 2.08 \text{ m s}^{-2} = a$$

$$\textcircled{1} \quad v = u + at$$

$$0 = 25 + \left(\frac{-625}{300}\right)(t)$$

$$\frac{25 \times 300}{625} = t = 12 \text{ seconds}$$

$$\textcircled{2} \quad s = ut + \frac{1}{2}at^2$$

$$150 = 25t + \left(\frac{-625}{600}\right)t^2$$

$$1.04t^2 - 25t + 150 = 0$$

$$t = 12 \text{ seconds}$$

$$b) \quad v = u + at$$

$$0 = 25 + 12a$$

$$\frac{-25}{12} = a = 2.1 \text{ m s}^{-2}$$

$$v = u + at$$

$$0 = 25 + 12a$$

$$\frac{25}{12} = a = 2.08 \text{ m s}^{-2}$$

$$= 2.1 \text{ m s}^{-2}$$

Projectile Motion

Falling Freely

When an object falls, it accelerates downwards. Force of Gravity acts on the object.

The object also pulls with a small force but the effect of the force on Earth is very small than the effect of Earth on the object.

The acceleration due to gravity at the Earth's surface varies at different place because Earth is not a perfectly perfect sphere and slightly flattened at the poles.

2) A cyclist ~~starts~~ slows uniformly a speed of 7.5 m s^{-1} to a speed of 2.5 m s^{-1} in a time of 5.0 s .

Calculate a) the acceleration
b) the distance moved in 5 sec

a) $v = u + at$

$$2.5 = 7.5 + 5a$$

$$-5 = 5a$$

$$a = -1.0 \text{ m s}^{-2}$$

b) $s = \left(\frac{v+u}{2} \right) t$

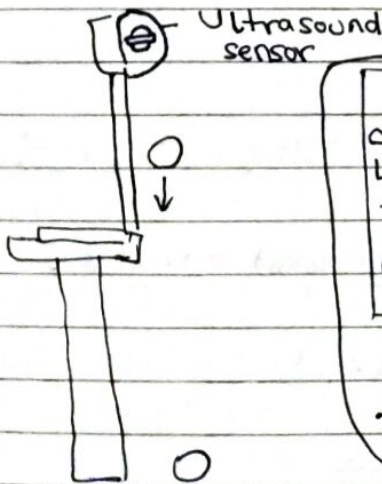
$$= \left(\frac{10.0}{2} \right) t$$

$$= 5 \times 5$$

$$= 25 \text{ m}$$

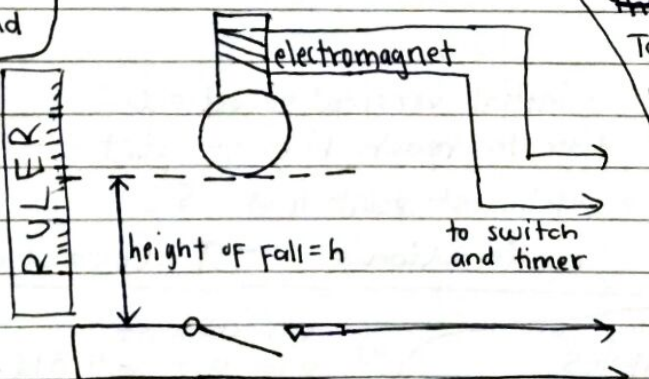
Experiments to measure g

Alternative 1



Data logger is used. The ultrasound sensor senses objects below it. Set the logger to measure speed at regular intervals. Object should be large enough. Switch on the system. The speed-time graph will be a straight line.

Alternative 2



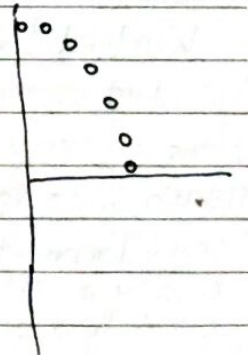
Magnetic field is produced by coiled wire which holds a ball. When current is switched off, the ~~ball~~ sphere falls. A clock starts. It stops when the ball opens the trapdoor and breaks the connection between the computer and terminals.

Calculate height.
Use $h = ut + \frac{1}{2}gt^2$
 $ut = 0$
 $h = \frac{1}{2}gt^2$

To reduce errors, change vertical distance and plot graph of h against t^2 .
Gradient = $\frac{g}{2}$

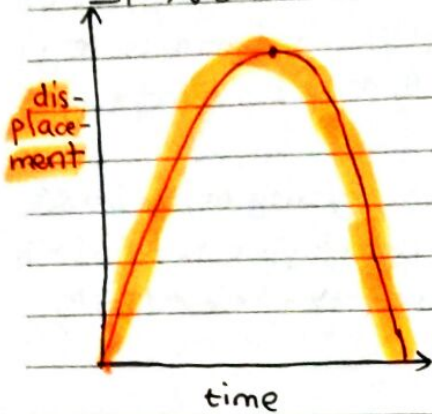
Alternative 3

There is a further. Take a video or multiframe image of a falling object and analyse the images to measure g.

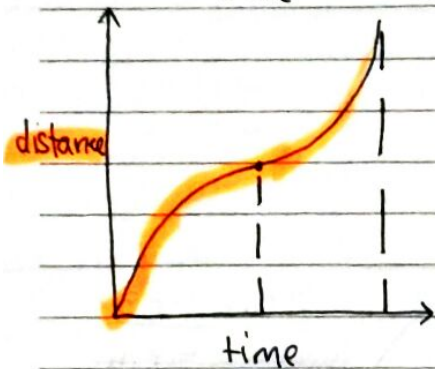


What goes up must come down

As a ball is released, ~~it must~~ force of gravity starts to take effect. The ball slows down, stops at the top of its motion and then falls back to Earth
IF NO AIR RESISTANCE



This is a graph of **Vertical Displacement**. Not the shape of the path or trajectory.



Distance-time graph is different. But, gives same information. Just that no direction.

SUVAT

initial vertical speed = u

time to reach highest point = t ($2t$ for whole motion)

highest point $h = s$

acceleration = $-g$ (because upward is positive direction)

Worked Examples

- 1) Student drops a stone from rest at the top of a well. She hears a splash 2.3s after releasing the stone. Ignore time of sound.
- Calculate depth of well.
 - Calculate speed at which the stone hits the water surface.
 - Explain why the time taken for the sound to reach the student can be ignored.

$$\begin{aligned} \text{a) } v &= u + at \\ v &= 0 + 9.81(2.3) \\ v &= 22.6 \text{ ms}^{-1} \end{aligned}$$

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ s &= 0 + \frac{1}{2}(9.81)(2.3)^2 \\ &= 25.9 \text{ metres} = 26 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{b) } v &= u + at \\ v &= 0 + 9.81(2.3) \\ &= 22.6 \text{ ms}^{-1} = 23 \text{ ms}^{-1} \end{aligned}$$

c) The distance between the water in well and the student is very small. Speed of sound is around 300 ms^{-1} and distance is 26 m . So, it took less than 0.1 sec for sound to reach the student. Only 4% of total time.

2) A hot air balloon is rising vertically at a constant speed of 5.0 m s^{-1} . A small object is released from rest relative to the balloon when the balloon is 30 m above the ground.

- a) Calculate the maximum height of the object above the ground.
 b) Calculate the time taken to reach the maximum height.
 c) Calculate the total time for the object to reach the ground.

a) $v^2 = u^2 + 2as$
 $0 = 25 + 2(-9.81)s$
 $\frac{-25}{-19.62} = s$

$s = 1.3 \text{ m}$
 $1.3 + 30 = 31.3 \text{ m}$

~~$v = u + at$~~
 ~~$s = \frac{(u+v)t}{2}$~~

~~$31.3 = \frac{0+5t}{2}$~~

~~$31.3 = 2.5t$~~

~~$\frac{31.3}{2.5} = t$~~

b) ~~$s = ut + \frac{1}{2}at^2$~~
 ~~$1.3 = 5t$~~

$v = u + at$

$0 = 5 + (-9.81)t$

$\frac{-5}{-9.81} = t$

$t = 0.51 \text{ seconds}$

c) $s = ut + \frac{1}{2}at^2$

$-31.3 = \frac{1}{2}(-9.81)(t^2)$

$6.38 = t^2$

$t = 2.53 \text{ seconds}$

$2.53 + 0.51 = 3.04$

$= 3.0 \text{ seconds}$

~~$t = 12.52$~~

~~$t = 13 \text{ seconds}$~~

Moving horizontally

Surface of Earth is large enough to be considered flat and there is no friction.

Horizontal acceleration = 0 m s^{-2} (No gravity acting horizontally)

• Horizontal velocity does not change

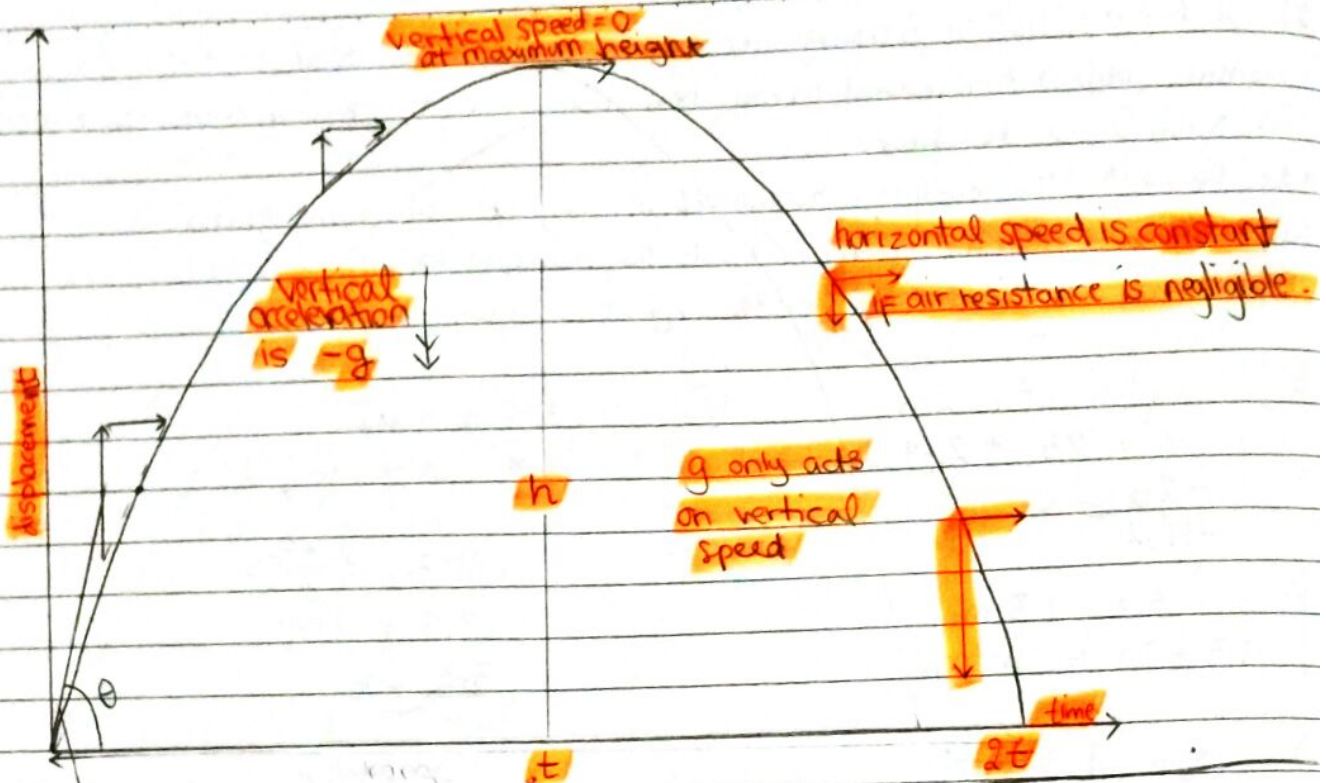
• Horizontal distance travelled = horizontal speed \times time for the motion.

$s \propto t^2$ IF $t = 2$, $s \rightarrow t$ doubles, s increasing by times 4.

Horizontal & Vertical motions are completely independent of each other.

Remember: the ^{Final} velocity is calculated by doing Pythagoras theorem of horizontal & vertical velocity.

Date



initial velocity, u

initial horizontal component = $u \cos \theta$

initial vertical component = $u \sin \theta$

at maximum height:

$$0 = u \sin \theta - gt$$

$$h = (u \sin \theta)t - \frac{gt^2}{2}$$

$$0 = u^2 \sin^2 \theta - 2gh$$

range = $2t(u \cos \theta)$

2 An object is thrown horizontally from a ship and strikes the sea 1.6s later at a distance of 37m from the ship.

- a) the initial horizontal speed of the object
 b) the height of the object above the sea when it was fired through.

a) $v = \frac{37}{1.6}$

$v = 23.125 \text{ ms}^{-1}$
 $= 23 \text{ ms}^{-1}$

b) $s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}at^2$

$s = \frac{1}{2}(9.81)(1.6^2)$

$s = 12.5568 \text{ m}$
 $= 12.6 \text{ m}$

KIKY

Worked Examples

1) An arrow is fired horizontally from the top of a tower 35m above the ground. The initial horizontal speed is 30 ms^{-1} . Assume air resistance is negligible. Calculate

- a) the time for which the arrow is in the air
 b) the distance from the foot of the tower at which the arrow strikes the ground
 c) the velocity at which the arrow strikes the ground

a) $s = ut + \frac{1}{2}at^2$

$35 = 0 + \frac{1}{2}(9.81)(t^2)$

$70 = 9.81t^2$

$t^2 = 7.14$

$t = 2.7 \text{ seconds}$

b) range = $t \times 30 \text{ ms}^{-1}$

$= 2.7 \times 30$

$= 81 \text{ metres}$

c) horizontal = 30 ms^{-1}
 vertical, $v = u + at$

$v = 0 + 2.7(9.81)$

$= 26.487 \text{ ms}^{-1}$

velocity = $\sqrt{(26.487^2 + 30^2)}$

$= 40 \text{ ms}^{-1}$

$\tan \theta = \frac{26.487}{30}$

$\theta = 41.40$

$= 41^\circ$

2.2 Forces

Date

Force is a push or pull.

Newton's First Law of motion

An object continues to remain stationary or to move at a constant velocity unless an external force acts on it.

Inertia - the ~~is~~ resistance to a change in ^{state of} motion of an object

Newton's Second Law

Force = Mass \times acceleration

$$F = ma$$

$$(N) \quad (kg)(m/s^2)$$

Inertial mass - property that permits an object to resist the effect of a force that is trying to change its motion.

Weight - property that arises from ^{the} gravitational attraction between the mass of an object and the mass of the Earth.

Inertial mass & gravitational mass are proportional.

Worked examples

1) A car with mass 1500 kg accelerates uniformly from rest to a speed of 28 m/s^1 (about 100 km/h^1) in a time of 11 s. Calculate average force that act on the car to produce this acceleration.

$$m = 1500 \text{ kg}$$

$$a = \frac{v-u}{t} = \frac{28}{11}$$

$$\begin{aligned} F &= ma \\ &= 1500 \times \frac{28}{11} \\ &= 1500 \times 2.54 \text{ N} \\ &= 3.8 \text{ kN} \end{aligned}$$

2) An aircraft of mass 3.3×10^5 takes off from rest in a distance of 1.7 km. Maximum thrust of the engines is 830 kN. Calculate take-off speed.

$$F = 830 \text{ kN} = 830000 \text{ N}$$

$$\frac{F}{m} = a = \frac{830000}{330000} = 2.5 \text{ m/s}^2$$

$$v^2 = u^2 + 2as$$

$$v^2 = 2(2.5)(1700)$$

$$v^2 = 8552$$

$$v = 92.547 \text{ m/s}^1$$

$$= 92.5 \text{ m/s}^1$$

Newton's third law

Every action has an equal and opposite reaction.

action force

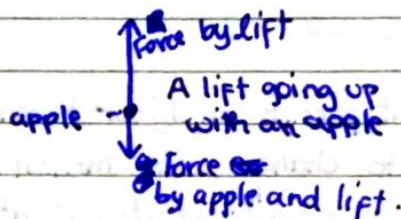
reaction force

action-reaction pair must be of the same type.

Free-body force diagrams

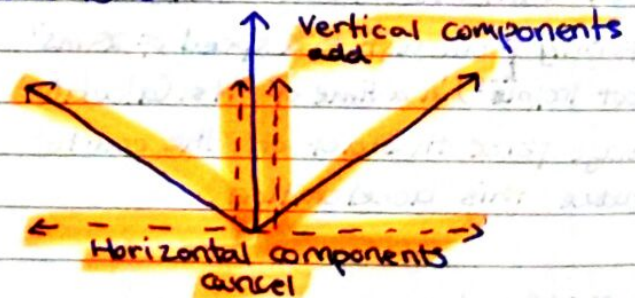
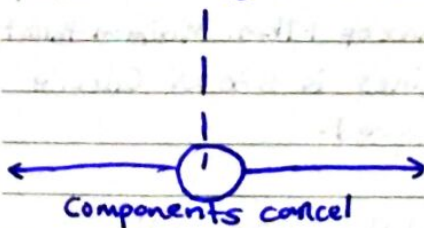
Rules

- One body only, force vectors are represented as arrows.
- Only forces acting on body are considered.
- Labelled forces



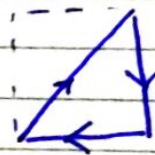
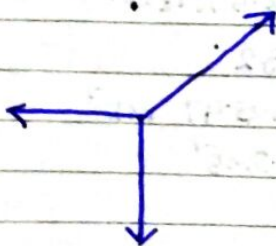
Translational equilibrium

An object at rest or moving at a constant velocity. Means it is moving in a straight line. Resultant force is 0.



Horizontal & Vertical components are independent of each other.

Equilibrium



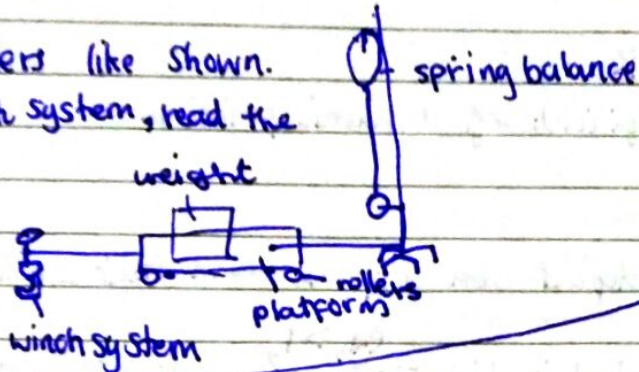
Triangle of forces can be made if system is in translational equilibrium

Solid friction

Friction is the force that occurs between two surfaces in contact.

Investigate

- Set up the platform on rollers like shown.
- Pull the platform using winch system, read the reading on balance.
- change weights, surface, lubrication etc



Static friction = when there is no relative movement between the surfaces

Dynamic friction = when there is relative movement

As pulling force ~~is experiment~~ increases but without slip happening, friction is static. When pulling force exceeds this value of static friction, the surfaces will start to move. As it moves, the friction moves to a new value which is lower than ^{than} the maximum ~~to~~ static friction. This is dynamic friction. Both frictions depend on the two surfaces concerned.

Static Friction

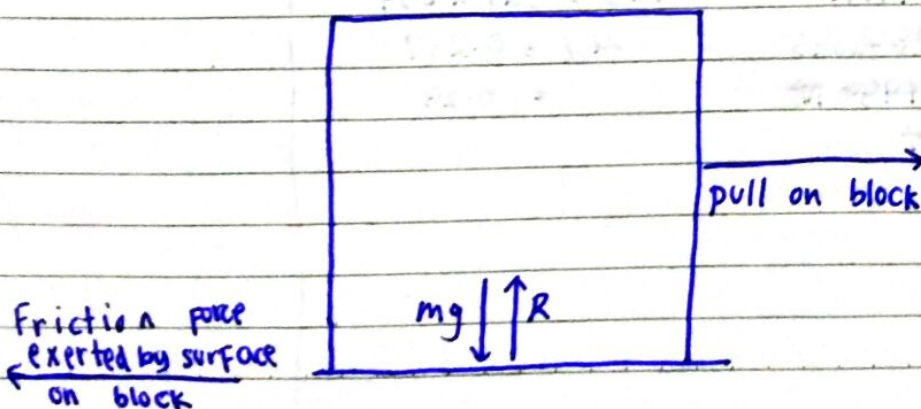
$$F_f \leq \mu_s R$$

μ_s = coefficient of static friction

F_f = frictional force

R = reaction force, same as weight.

The static friction can be from 0 to maximum value (\leq). Once the pull on block is equal to $\mu_s R$, then the block starts to move. Then the friction operates in the dynamic regime.



Dynamic Friction

Applied when surfaces move relative to each other. The friction drops from maximum and remains constant. Depends on Reaction force, not relative speed between the surfaces.

$$F_f = \mu_d R$$

μ_d = coefficient of dynamic friction

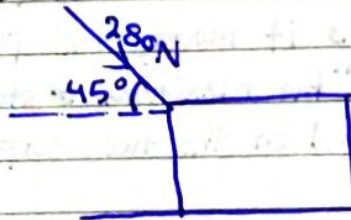
μ_s & μ_d depend on surfaces and their condition (e.g. lubricated or not).

If coefficient is high for ^{eg, > 1,} friction is very strong and greater than the weight of the block.

Worked Examples

1) A box is pushed across a level floor at a constant speed with a force of 280 N at 45° to the floor. The mass of the box is 50 kg.

Calculate



- vertical component of force
- weight of box
- horizontal component of ^{Force} box
- coefficient of dynamic friction between the box and the floor

a. $280 \sin 45$
 $= 197.9 \text{ N}$
 $= 198 \text{ N}$

~~200 N~~

c. $280 \cos 45 = 198 \text{ N}$

d. $50 \times 9.81 = 491 \text{ N}$
 $491 + 198 = 689 \text{ N}$

Reaction force = 689 N

b. 50×9.81
 $= 490.5 \text{ N}$

~~490 N~~ = 491 N

$F_f = 198 \text{ N}$

$F_f = \mu_d \times R$

$198 = \mu_d \times 689$

$\mu_d = 0.287$

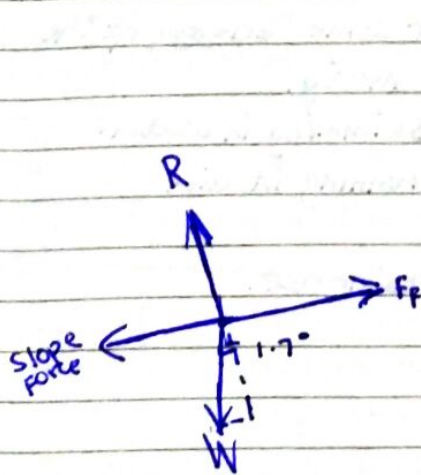
$= 0.29$

~~$280 \cos 45$~~ ~~$280 \sin 45$~~
 ~~$= 200 \cos 45 + 198 \text{ N}$~~

~~$= 200 \text{ N}$~~

2. A skier places a pair of skis on a snow slope that is at an angle of 1.7° to the horizontal. The coefficient of static friction between the skis and the snow is 0.025.

Determine whether the skis will slide away by themselves.



$$\mu_s = 0.025$$

Weight is the independent force with v and h components
Imagine as if axis is slant

$$R = W \cos 1.7^\circ$$

$$F_f = W \sin 1.7^\circ$$

$$F_f \leq \mu R$$

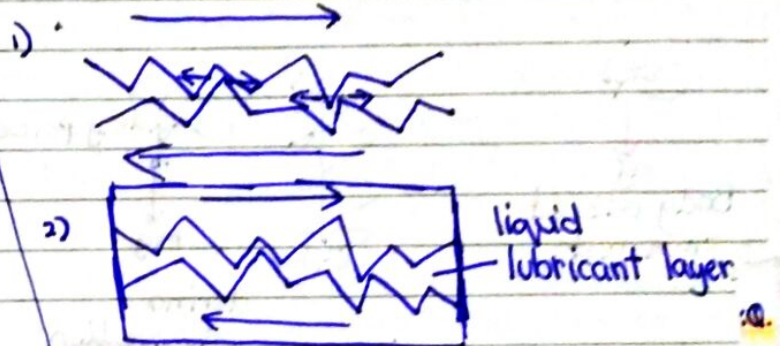
$$W \sin 1.7 \leq 0.025 W \cos 1.7$$

$$\frac{\sin 1.7}{\cos 1.7} \leq 0.025$$

$$0.0297 \leq 0.025 \quad \times$$

The skis will slide away because the coefficient is higher than the maximum static friction coefficient.

Origins of Friction



Surfaces are not smooth and have peak and troughs of atoms.

When static friction occurs and two surfaces are at rest relative to each other then the atomic peaks rest in the trough.

and it needs a certain level of force to deform or break the peak sufficiently sliding to begin. Once the relative motion has started, the peak to p surface rises a little above the deformed peaks. Less force required.

Lubricants get between the spaces. Either prevents the peaks & troughs of atoms from touching or reduces the amount of contact. Not as much interaction between atoms and therefore, coefficient decreases.

The friction equations are empirical not ^{theoretical} experimental. They ^{are} derived from experiments.

Fluid resistance and terminal speed

An object travelling through a fluid (liquid or gas) experiences drag force.

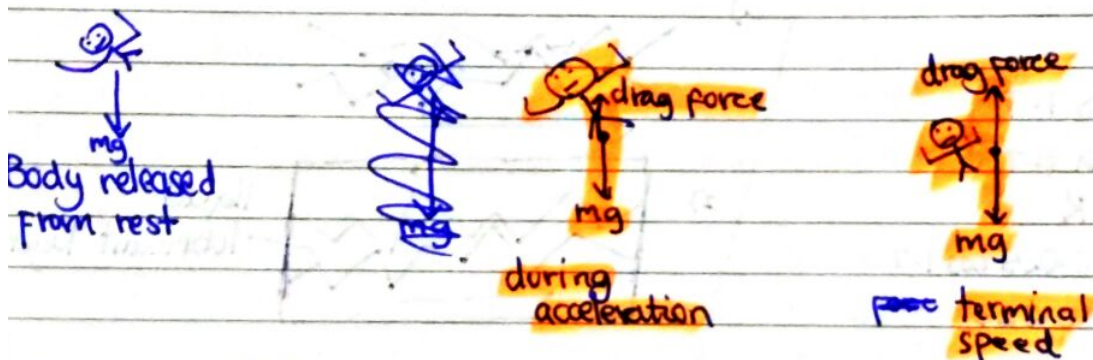
Air resistance -

the action of air resistance is to transfer some energy of the moving body to the fluid through which it is moving.

Some fluids absorb more energy than others: Swimming in water is more tiring than ~~swimming in water~~ air running in air.

In IB, need to describe the effects of fluid resistance.

Sky diving



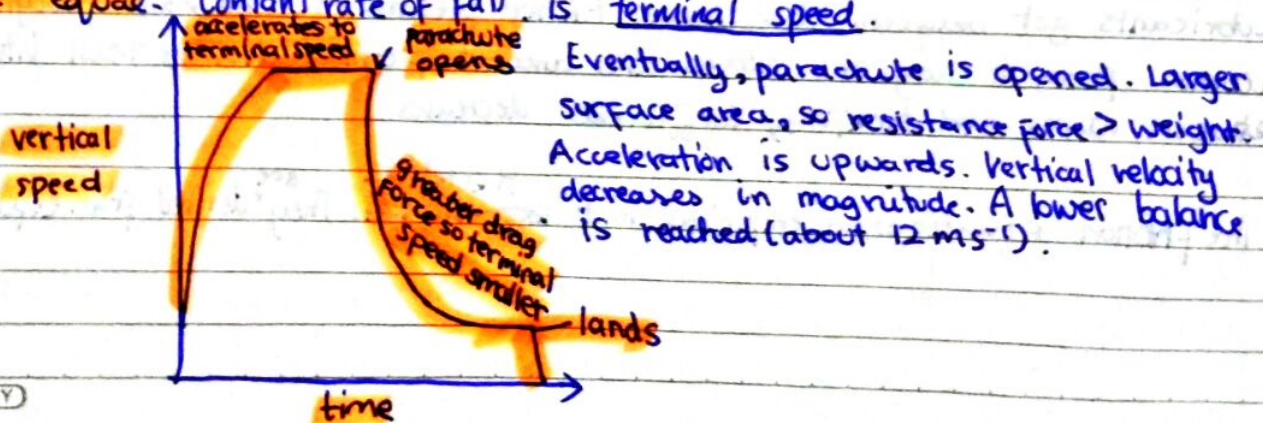
approx.

Weight is constant because there are little changes in gravitational field strength at the height of dive.

Air resistance acts vertically, upwards.

(Other force - upwards buoyancy caused by displacement of air by the diver.)

When skydiver jumps, $v = 0$, so no air resistance. As speed increases, air resistance increases. Net force therefore decreases and the acceleration decreases. Eventually weight force and resistance force are equal. Constant rate of fall is terminal speed.



Eventually, parachute is opened. Larger surface area, so resistance force $>$ weight. Acceleration is upwards. Vertical velocity decreases in magnitude. A lower balance is reached (about 12 m s^{-1}).

Maximum speed of a car

1. As speed increases, drag force of a ~~car~~ car increases. Typically when speed doubles, the drag force will increase by at least a factor of four.

Engine has a capacity. Has a maximum power. When speed increases towards maximum, power dissipated in friction also increases. When maximum energy output of the engine every second is completely used in overcoming the energy losses, then car has reached its maximum speed.

Worked example

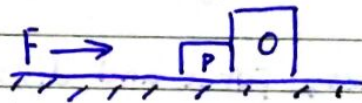
1. Calculate the upward force acting on a skydiver of mass 80 kg who is falling at a constant speed.

$$\begin{aligned} \text{Weight} &= 80 \times 9.81 = 784.8 \text{ N} \\ &= 785 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Weight} &= \text{Upward force} \\ &= 785 \text{ N} \end{aligned}$$

2. ~~P & Q~~

2. P & Q are two boxes that are pushed across a rough surface at a constant velocity with a horizontal force of 30 N. The mass of P is 2.0 kg and the mass of Q is 4.0 kg.



State the resultant force on box Q

Since constant velocity,
resultant force = 0 N

2.3 Work, Energy and Power

Kinetic energy is associated with the motion of a mass.

Potential energy is associated with the position of a mass in a gravitational field.

One joule is the energy required when a force of one newton acts through a distance of one metre.

Principle of conservation of energy - energy cannot be created or destroyed.

$$1 \text{ kJ} = 10^3 \text{ J}$$

$$1 \text{ MJ} = 10^6 \text{ J}$$

$$1 \text{ GJ} = 10^9 \text{ J}$$

$$1 \text{ calorie} = 4.2 \text{ J}$$

Worked Example

Describe the mechanisms associated with the energy changes that occur when a balloon is blown up.

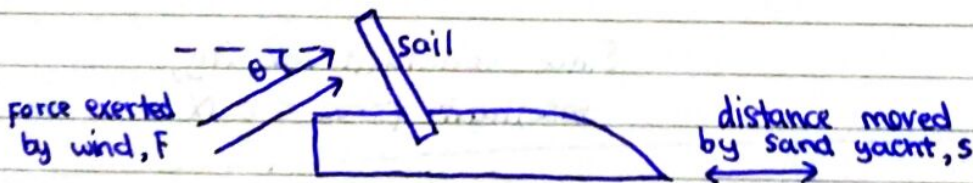
Air molecules gain kinetic energy that is used to store elastic potential energy in the balloon's skin and to make changes in the energy of the air. The air inside is able to exert more force (pressure) outwards on the skin of the balloon until a new equilibrium is established between the tension in the skin and the atmospheric pressure.

Doing Work

Work done (J) = Force exerted (N) \times distance moved in the direction of force (m)

$$W = Fd$$

Sometimes, force and distance are not in the same direction.
E.g. A yacht



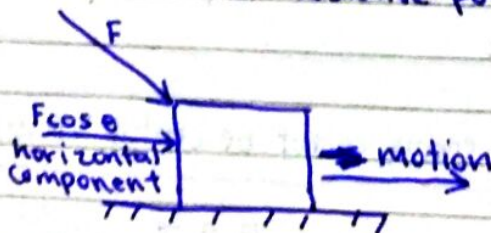
Horizontal component of force = $F \cos \theta$

$$\text{Workdone} = F \cos \theta \times s$$

$$= F \cos \theta \times s$$

Work done against a resistive force

Work is done when a resistive force is operating too.

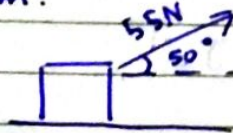


Worked examples

1. The thrust of a microlight aircraft engine is $3.5 \times 10^3 \text{ N}$. Calculate the work done by the thrust when the aircraft travels a distance of 15 km.

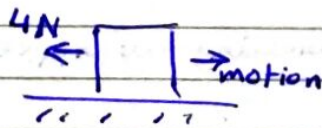
$$\begin{aligned} W &= fd \\ &= (3.5 \times 10^3)(15000) \\ &= 52500000 \text{ J} \\ &= 5.25 \times 10^7 \text{ J} \\ &= 53 \text{ MJ} \end{aligned}$$

2. A box is pulled a distance of 8.5 m by a force of 55 N on a rough surface. Force is 50° to the horizontal. Calculate work done in moving the box 8.5 m.



$$\begin{aligned} \text{Horizontal component} &= 55 \cos 50^\circ \\ W &= 55 \cos 50^\circ \times 8.5 \\ &= 300.5 \text{ J} \\ &= 301 \text{ J} \end{aligned}$$

3. Object moving in a straight line has an initial $K_E 24 \text{ J}$. Calculate distance in which object will come to rest if a 4.0 N net force, opposes the motion.



$K_E = \text{Energy}$

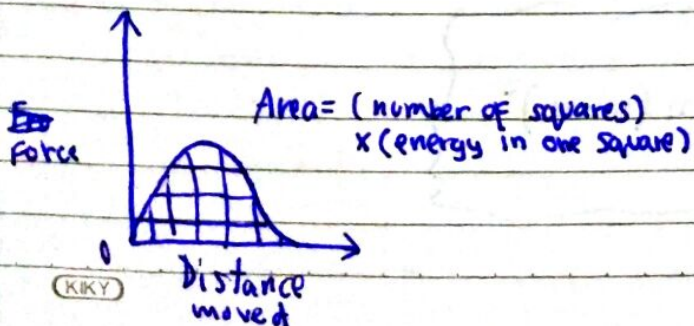
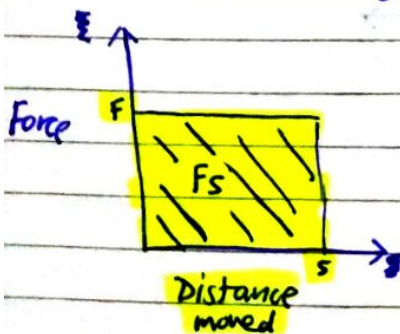
24 J transformed

24 = fd because 24 J work must be done to make $K_E = 0$...

$$24 = 4d$$

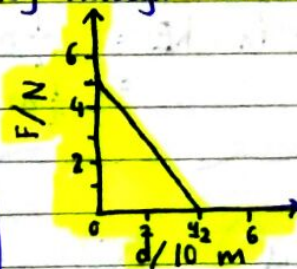
$$d = 6 \text{ m}$$

Force-distance graphs



Worked example

The graph shows the variation with displacement d of a force F that is applied to a toy car. Calculate work done by F in moving the toy through a distance of 4.0 cm.



$$\begin{aligned} \frac{1}{2} \times 5 \times 4 &= 20 \text{ J} \times \frac{1}{2} = 10 \text{ J} \\ 10 \times 10^{-2} &= 0.10 \text{ J} \end{aligned}$$

Power

Power = rate of doing work

Number of joules being converted every second.

$$= \frac{\text{Energy Transformed}}{\text{Time taken for transfer}}$$

$$1 \text{ W} = 1 \text{ J s}^{-1}$$

Another useful expression:

$$P = \frac{W}{t}$$

$$= \frac{F d}{t}$$

$$= F \times \frac{d}{t}$$

$$= \text{Force} \times \text{speed}$$

$$P = Fv$$

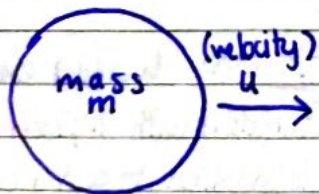
Power required to move an object travelling at a speed v with a force F is Fv .

$$1 \text{ horsepower} = 750 \text{ W}$$

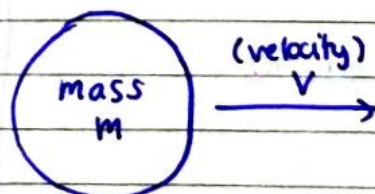
Kinetic Energy

Caused by motion, E_k .

Objects gain E_k when their speed increases.



$$KE = \frac{1}{2} m u^2$$



$$KE = \frac{1}{2} m v^2$$

$$\begin{aligned} \text{change in } KE &= \frac{1}{2} m v^2 - \frac{1}{2} m u^2 \\ &= \frac{1}{2} m (v^2 - u^2) \end{aligned}$$

TIP:

$$\frac{1}{2} m (v^2 - u^2) \neq \frac{1}{2} m (v - u)^2$$

Worked Examples

- 1) A car is travelling at constant speed of 25 ms^{-1} and its engine is producing a useful power output of 20 kW . Calculate driving force required to maintain this speed.

$$P = Fv$$

$$20 \text{ kW} = F \times 25$$

$$20000 = 25F$$

$$800 = F$$

$$F = 800 \text{ N}$$

- 2) A vehicle is being designed to capture the world land speed record. It has a maximum design speed 1700 kmh^{-1} and a fully fuelled mass 7800 kg . Calculate maximum kinetic energy.

$$\begin{aligned} E_k &= \frac{1}{2} (7800) \left(\frac{1700000}{3600} \right)^2 \\ &= \frac{1}{2} (7800) (222993.8272) \\ &= 86967.5925.9 \text{ J} \\ &= 869676 \text{ kJ} \\ &= 870 \text{ MJ} \\ &= 0.87 \text{ GJ} \end{aligned}$$

- 3) A car of mass $1.3 \times 10^3 \text{ kg}$ accelerates from a speed of 12 ms^{-1} to a speed of 20 ms^{-1} . Calculate the change in KE of the car.

$$\begin{aligned} \Delta KE &= \frac{1}{2} (1.3 \times 10^3) (20^2 - 12^2) \\ &= 650 (256) \end{aligned}$$

$$\begin{aligned} \Delta KE &= 166400 \text{ J} \\ &= 1.7 \times 10^5 \text{ J} \end{aligned}$$

~~0.17 MJ~~

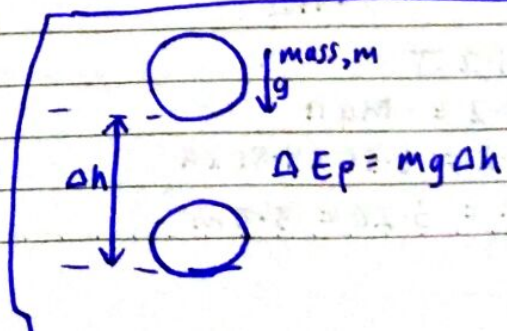
Gravitational potential energy, GPE

Energy of an object due to its position in the gravitational field.

In this, $F = mg$ and $d = \Delta h$

$$W = mg\Delta h$$

$$\Delta E_p = mg\Delta h$$



Energy moving between GPE and KE

Friction, air resistance - negligible

Conservation of energy

$$\text{Maximum } E_p = \text{Maximum } E_k$$

$$\Delta E_p = -\Delta E_k$$

Suppose, a snowboarder is going down from rest.

Vertical change in height of slope = 50 m

What is the speed at bottom of the slope?

$$\frac{1}{2} E_p = \frac{1}{2} mv^2$$

$$= 50 mg = \frac{1}{2} mv^2$$

$$= 50 \times 9.81 \frac{m}{s^2} = \frac{1}{2} mv^2$$

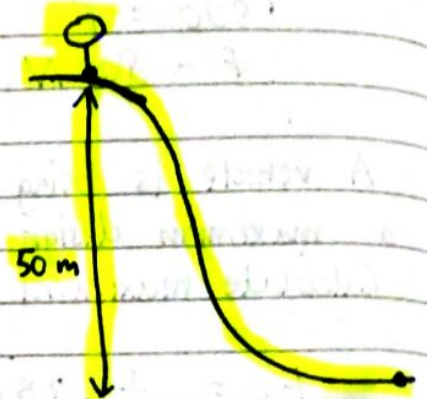
$$490.5 \frac{m}{s^2} = \frac{1}{2} mv^2, \text{ Mass cancels out.}$$

$$490.5 = \frac{1}{2} v^2$$

$$981 = v^2$$

$$v = 31.3 \text{ m s}^{-1}$$

$$= 31 \text{ m s}^{-1}$$



Worked examples

1. A ball of mass 0.35 kg is thrown vertically upwards at a speed of 8.0 m/s. Calculate -

a. initial kinetic energy.

b. maximum gravitational potential energy.

c. maximum height reached.

a. $\frac{1}{2} mv^2 = E_k$

$$\frac{1}{2} (0.35)(8^2) = 11.2 \text{ J}$$

b. 11.2 J

c. $11.2 = mgh$

$$11.2 = 0.35 \times 9.81 \times h$$

$$h = 3.26 = 3.3 \text{ m}$$

2. A pendulum bob is released from rest 0.15 m above its rest position. Calculate the speed at it passes through the rest position.

$$\Delta h = 0.15 \text{ m}$$

$$E_p = 1.4715 \text{ m}$$

$$\frac{1}{2}mv^2 = 1.4715 \text{ m}$$

$$\frac{1}{2}v^2 = 1.4715$$

$$v^2 = 2.943$$

$$v = 1.72 \text{ m s}^{-1}$$

$$\approx 1.7 \text{ m s}^{-1}$$

Elastic potential energy

Different materials respond differently to forces.

Some materials are able to return the energy when the force is removed. Eg. Spring

Materials that can return energy in this way stored Elastic potential energy.

Hooke's Law

For small loads acting on a spring, Hooke showed that extension of the spring is directly proportional to the load.

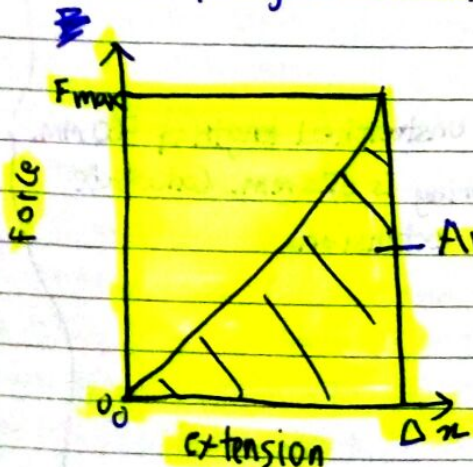
This means, graph of F against (Δx) is a straight line going through the origin.

$$F \propto \Delta x$$

Gradient of the graph shows the value of k .

$$F = k\Delta x$$

k = spring constant, Nm^{-1}



$$\text{Area} = \frac{1}{2} F_{\text{max}} \times \Delta x$$

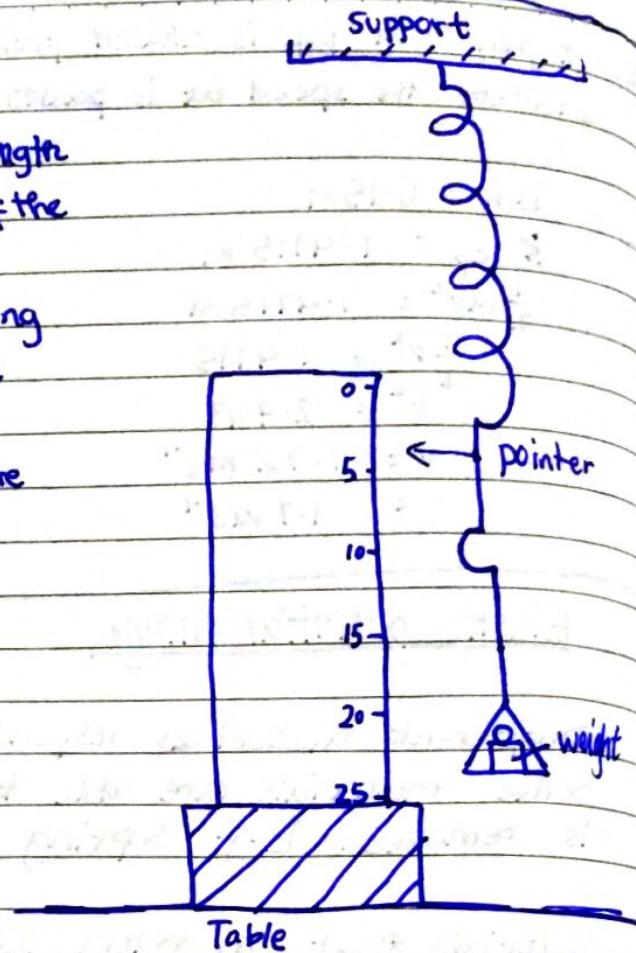
= work done in extending spring

= stored elastic potential energy

$$E_p = \frac{1}{2} F \Delta x \text{ or } \frac{1}{2} k (\Delta x)^2$$

Investigating Hooke's Law

- Arrange a spring of known unstretched length with a weight hanging on the end of the spring.
- Measure the extensions for increasing weights hanging on the end of the spring.
- Repeat the measurements as you remove the weights as a check.
- Plot a force-extension graph.
y-axis x-axis



Worked Examples

- 1) A spring, of spring constant 48 Nm^{-1} , is extended by 0.40 m . Calculate E_p stored in the spring.

$$\begin{aligned}
 E_p &= \frac{1}{2} F \Delta x \\
 &= \frac{1}{2} k (\Delta x)^2 \\
 &= \frac{1}{2} \times 48 \times 0.40^2 \\
 &= 24 \times 0.16 \\
 &= 3.84 \text{ J}
 \end{aligned}$$

- 2) An object of mass 0.78 kg is attached to a spring of unstretched length of 560 mm . When the object has come to rest the new length of the spring is 620 mm . Calculate the energy stored in the spring as a result of the extension.

$$620 - 560 = 60 \text{ mm} = \Delta x$$

$$\frac{60}{10} = 6 \text{ cm} \quad \frac{6}{100} = 0.06 \text{ m}$$

$$E_p = \frac{1}{2} k (\Delta x)^2 \text{ or } \frac{1}{2} F \Delta x$$

$$(KIKY) = \frac{1}{2} (0.78 \times 9.81) (0.06)$$

$$E_p = 0.2296 = 0.23 \text{ J}$$

Efficiency

Some energy is lost due to friction, sound, heat etc.

We can quantify these losses. - By comparing the total energy input and useful energy output. This is known as efficiency of the transfer. This can be applied to all energy transfers - mechanical, electrical or anything.

Efficiency calculations are everywhere where there are energy transfers.

$$\text{Efficiency} = \frac{\text{Useful work out}}{\text{total energy in}} = \frac{\text{Useful power out}}{\text{total power}}$$

Worked example

1) An electrical motor raises a weight of 150 N through a height of 7.2 m. The energy supplied to the motor during this process is 3.5×10^4 J.

$$\begin{aligned} \text{a. } \Delta E_p &= mg\Delta h \\ &= 150 \times 7.2 \\ &= 1080 \text{ J} \end{aligned}$$

Calculate:

$$\text{b. } 3.5 \times 10^4 \text{ J} = 35000 \text{ J}$$

- a. the increase in gravitational potential energy
b. the efficiency in the process

$$\begin{aligned} 100 \times \frac{1080}{35000} &= 3.09\% \\ &= 3.1\% \end{aligned}$$

24 MOMENTUM

Catching a table tennis and baseball at the same speed! One may be a more painful experience than the other. Difference in the objects is mass. If catching a firm throw and a powerful hit from a professional player, you will know that velocity makes a difference too.

Momentum = mass x velocity (not speed)

$$\begin{aligned} p &= mv \\ (\text{kgms}^{-1}) & \quad (\text{kg} \times \text{ms}^{-1}) \end{aligned}$$

Momentum has direction.

Worked Examples

2) A ball of mass 0.25 kg is moving to the right at a speed of 7.4 ms^{-1} . It strikes a wall at 90° and rebounds from the wall leaving it with a speed of 5.8 ms^{-1} moving to the left. Calculate the change in momentum.

$$0.25 \times 7.4 = 1.85 \text{ kg ms}^{-1} \text{ to right}$$

$$0.25 \times 5.8 = 1.45 \text{ kg ms}^{-1} \text{ to left}$$

$$\cancel{\Delta p = 1.85 - (-1.45) = 0.40 \text{ kgms}^{-1}}$$

$$\Delta p = 0.40 \text{ kgms}^{-1}$$

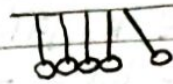
$$1.85 - (-1.45) = 3.3 \text{ kgms}^{-1}$$

$$\Delta p = 3.3 \text{ kgms}^{-1} \text{ to the left}$$

$$\text{OR } -3.3 \text{ kgms}^{-1} \text{ to the right}$$

Collisions and changing momentum

Analysing Newton's cradle.



Right

As ~~right~~ right-hand ball ~~is~~ gains ~~the~~ GPE when raised. The GPE turns to KE when ball is released. Eventually the ball strikes the next stationary ball, and action-reaction forces act ~~at~~ at surfaces of the two balls. The stationary ball is compressed slightly and ~~it moves the compression~~ the compression moves along as a wave through the middle three balls. When compression reaches the left-hand ball, elastic potential energy changes to KE and ~~the~~ work is done against gravity. ~~When~~ When speed = 0, motion repeats in the reverse direction.

If only two balls, can view this as a transfer of momentum.
This is a transfer of momentum

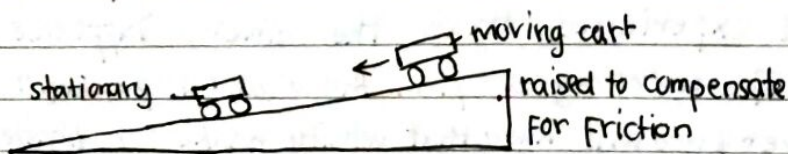


As the right-hand ball falls, it gains momentum. When it collides with the left-hand sphere, momentum is transferred. The first sphere now has 0 momentum and second has gained momentum.

Collision - any interaction where momentum is transferred.

E.g. Newton's cradle, bat hitting ball, firing a gun

Experiment for checking if momentum is conserved



- ~~Get~~ Measured masses ^{before hand}

- ~~Measure~~ Measure velocity
 - Data logger & sensor
 - Stop watch & distance

- Find initial & final momentum

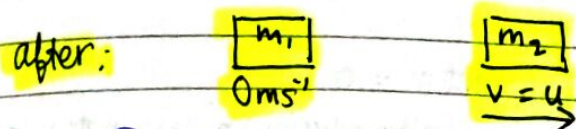
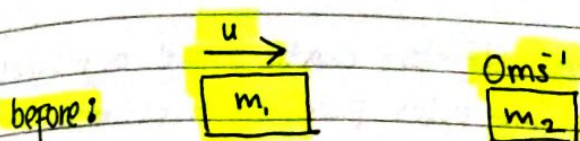
Momentum is always constant if no external force acts on the system.

This is the principle of conservation of linear momentum.

"Linear" because this is momentum when objects are moving in straight lines.

Momentum conservation is important. So, let's consider a few situations.

Two objects with the same mass, one initially stationary, when no energy is lost



$$m_1 u = m_2 v$$

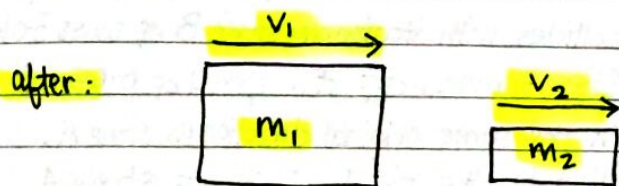
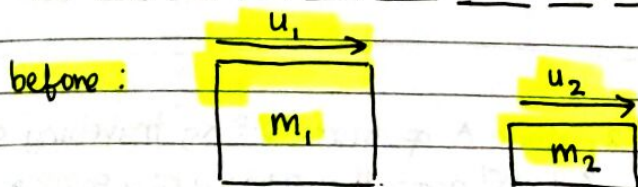
Initial velocity of m_1 is equal to the final velocity of m_2
Kinetic energy also remains same.

This is an elastic collision.

- No permanent deformation in colliding objects
- No internal energy is released (friction, sound or anything)

- Kinetic energy before & after remains same. $\frac{1}{2} m_1 u^2 = \frac{1}{2} m_2 v^2$

Two objects with different masses when no energy is lost



This is also an elastic collision.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

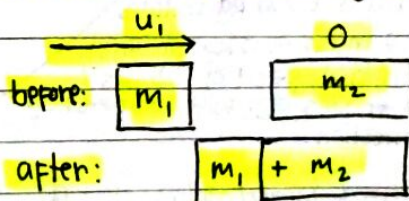
$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Two objects colliding when energy is lost

When a moving object collides with a stationary one and the objects stick together, some of the initial kinetic energy is lost. Finally, there is one object with an increased mass and a single common velocity.

This is an inelastic collision.

- Objects form a single object
- Mass increases
- ~~One~~ ^{Single} common velocity
- Kinetic energy is not conserved



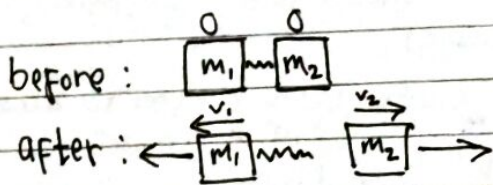
$$m_1 u_1 = (m_1 + m_2) v_1$$

$$v_1 = \frac{m_1 u_1}{(m_1 + m_2)} \text{ OR } \frac{m_1}{m_1 + m_2} u_1$$

The ratio $\frac{\text{initial kinetic energy}}{\text{Final kinetic energy}} = \frac{(m_1 + m_2)}{m_1}$

Two objects when energy is gained

There are many occasions like this. E.g. two objects - carts have a magnet attached to the front of the carts. The like poles face each other. When they are released after being stuck together, the magnets repel and the carts drive apart.



Initial momentum = 0

Final momentum (after collision), $m_1 v_1 + m_2 v_2 = 0$

So, $m_1 v_1 = -m_2 v_2$

If masses are equal, $v_1 = -v_2$

If masses are not equal, $\frac{m_1}{m_2} = -\frac{v_2}{v_1}$

Worked examples

1. A rail truck of mass 4500 kg moving at a speed of 1.8 m s^{-1} collides with a stationary truck of mass 1500 kg. The two trucks couple together. Calculate the speed of the truck immediately after the collision.

Initial momentum

$$(4500 \times 1.8) + 0 = 8100 \text{ kgms}^{-1}$$

Final momentum

$$8100 \text{ kgms}^{-1}$$

$$8100 = (4500 + 1500)v$$

$$8100 = 6000v$$

$$v = 1.35 \text{ m s}^{-1} = 1.4 \text{ m s}^{-1}$$

2. Stone A of mass 0.5 kg travelling at 3.8 m s^{-1} across the surface of a frozen pond collides with stationary stone B of mass 3.0 kg. Stone B moves off at a speed of 0.65 m s^{-1} in the same original direction as stone A. Calculate the final velocity of stone A.

Initial momentum

$$(0.5 \times 3.8) + 0 = 1.9 \text{ kgms}^{-1}$$

Final momentum

$$1.9 \text{ kgms}^{-1}$$

$$1.9 = 0.5v_1 + (3 \times 0.65)$$

$$1.9 = 0.5v_1 + 1.95$$

$$-0.05 = 0.5v_1$$

$$v_1 = -0.1 \text{ m s}^{-1}$$

0.10 m s^{-1} to the opposite direction of original motion.

3) A railway truck of mass 6000 kg collides with a stationary truck of mass 3000 kg. The first truck moves with an initial speed of 2.5 m s^{-1} and the second truck moves off with a speed of 1.9 m s^{-1} in the same direction.

Calculate

a. the velocity of the first truck immediately after the collision

b. the loss in kinetic energy as a result of the collision

$$a. (6000 \times 2.5) = 6000v + (3000 \times 1.9)$$

$$15000 = 6000v + 5700$$

$$9300 = 6000v$$

$$v = 1.55 \text{ m s}^{-1} = 1.6 \text{ m s}^{-1}$$

$$b. \frac{1}{2}(6000)(2.5^2) = 18750 \text{ J}$$

$$\frac{1}{2}(6000)(1.55^2) + \frac{1}{2}(3000)(1.9^2) = 12622.5 \text{ J}$$

$$18750 \text{ J} - 12622.5 \text{ J} = 6127.5 \text{ J}$$

$$= 6100 \text{ J}$$

Energy and Momentum

$$E_k = \frac{1}{2} mu^2$$

$$\text{momentum, } p = mu$$

$$p^2 = m^2 u^2$$

$$E_k = \frac{m^2 u^2}{2m}$$

$$= \frac{p^2}{2m}$$

Applications of momentum conservation

~~Recoil gun~~

Recoil of a gun

A gun (cannon) can be fired to trigger a snow fall. This prevents a more dangerous avalanche (rapid snow falling).

When the gun fires its shell, the gun moves backwards in the opposite direction to which the shell goes.

Explanation in terms of momentum conservation.

Initially, both gun and shell are stationary; the initial total momentum is zero.

The shell is propelled in the forward direction through the gun by the expansion of gas following the detonation of the explosive in the shell.

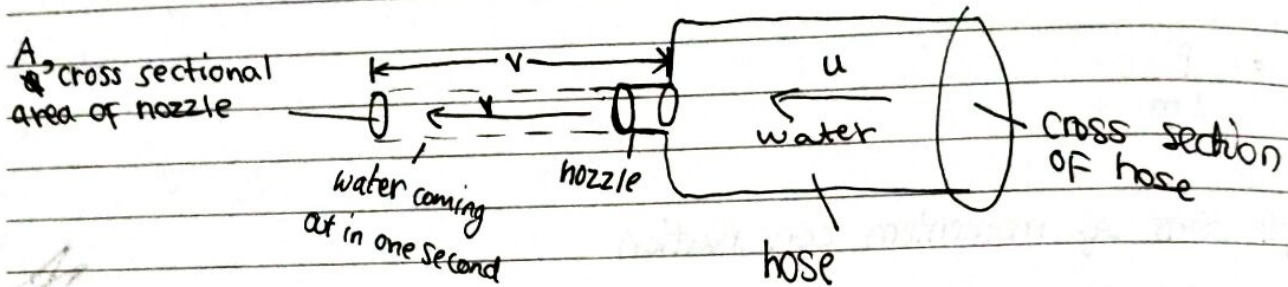
The explosion is an internal force in the system. Gas exerts a force on the interior of the shell chamber and hence a force ^{on} the gun is also applied. The explosive releases energy and this is transferred into kinetic energies of both the shell and gun.

The initial p was 0 and no external force acted on the system. The momentum ~~at the end~~ must be 0 throughout, so ~~the~~ the gun and the shell move in opposite directions with the same magnitude of momentum. The shell is much faster because it has a much smaller ^{er} mass compared to the gun; the gun moves relatively slowly.

Water hoses

A fire being extinguished by firemen using a high-pressure hose is a perfect example of water leaving the system.

Two or more firemen control the hose because there is a large force in the opposite direction to that of the water. You can see this when a garden hose starts to shoot backwards unpredictably if it is not held when the tap is turned on.



The cross-sectional area of the hose is greater than the nozzle. The mass of water flowing past a point in the hose every second is the same as the mass that emerges from the nozzle every second. So the speed of the water emerging from the nozzle is higher than the speed of flow of water in the hose. So, the water gains momentum as it leaves the hose since exit speed $>$ flow speed, and mass is same.

Momentum must be constant, so there must be a backward force on the end of the hose which the firemen must counter. The E_k and momentum are supplied by the water pump that feeds water to the fire hose or whatever creates the pressure in the garden tap.

(not really) $m =$ mass of water leaving every second

$$\text{Momentum lost every second (by the system)} = [m \times v (\text{leaving nozzle})] - [m \times v (\text{in hose})]$$

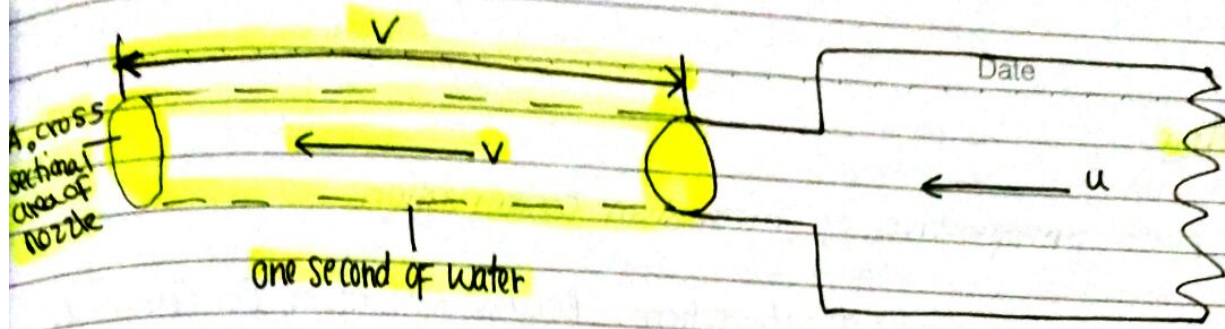
$$= \cancel{mv} - \cancel{mv} = mv - mu$$

$$\text{Momentum lost every second} = m(v - u)$$

$$\text{Mass of water lost per second} = \frac{\Delta m}{\Delta t}$$

$$\text{So, momentum lost per second (by the system)} = \frac{\Delta m}{\Delta t} (v - u)$$

So its like, momentum lost per second = mass lost per second \times velocity change per second



If we know cross-sectional area A and density of water, ρ , then $\frac{\Delta m}{\Delta t}$ can be determined.

Every second, you can imagine a cylinder of water leaving the hose during a one second ~~time~~ time interval; cylinder is v ^{m/s} long and has a cross-sectional area of A . So, volume of water leaving per second is Av .
(Mass of water leaving per second)

$$\frac{\Delta m}{\Delta t} = \text{density} \times Av = \rho Av$$

$$\frac{\Delta m}{\Delta t} = \rho Av$$

Since, mass entering and leaving the nozzle ^{per second} is the same, this means the change in momentum ~~is~~ in one second = $\frac{\Delta m}{\Delta t} (v-u) = \rho Av(v-u)$

If $u \ll v$, then expression simplifies as ~~ρAv^2~~ ~~ρAv^2~~ ρAv^2

Change in momentum per second

$$\frac{\Delta m}{\Delta t} = \rho Av^2 \quad \text{momentum}$$

$$\frac{\Delta m}{\Delta t} = \rho Av^2 \quad \frac{\Delta m}{\Delta t} = \rho Av^2 \quad \text{density (rho)}$$

The hose is an example where care is needed. E.g. if the water is directed at a vertical wall, the water strikes, loses all horizontal momentum, flows down the wall vertically. The momentum must have gone into the wall, its foundations, and, therefore, the ground. So, conclusively earth gained momentum and we can speed up Earth's rotation using a garden hose. But, why isn't this true? Because momentum was given originally by a pump and the gain in momentum at the pump must have give some momentum to Earth too. The amount of momentum the Earth gained at the pump is equal & opposite to the momentum gained by the Earth when the water strikes the wall.

$$\Delta P = \frac{\Delta m}{\Delta t} (v-u) = \rho Av^2$$

Rocketry

Rockets from perspective of momentum conservation.

Rockets are effective in absence of atmosphere. Because Momentum is conserved.

All rockets release a fluid at high speed.

The fluid escapes from ~~the~~ the combustion/storage chamber through nozzles at the base of the rocket. So, it accelerates in the opposite direction to the direction in which the fuel is ejected. Momentum is conserved. The rate of loss of momentum from the rocket in form of high speed ^{fluid} v is equal to the rate of gain in momentum of the rocket.

Helicopters

These can takeoff and land vertically. Also can hover motionless above a point on the ground.

Helicopter uses the principle of conservation of ^{linear} momentum in order to hover. The rotating blades exert a force on originally stationary air causing it to move downwards towards the ground gaining momentum in the process. No external force acts and as a result there is an upward force on the helicopter through the rotors.

Impulse

$F = ma$ is Newton's second law of motion

We can rearrange since $a = \frac{(v-u)}{t}$

$$F = \cancel{m} \frac{(v-u)}{t} m \times \frac{(v-u)}{t}$$

$$F = \frac{m(v-u)}{t}$$

Which means that force = $\frac{\text{change in momentum}}{\text{time taken for change}}$

OR

$$\text{force} \times \text{time} = \text{change in momentum} \quad (\text{Ns})$$

The equation shows that we can change momentum (accelerate an object) by exerting a large force for a short time or by exerting a small force for a long time. E.g. A small number of people can get a heavy vehicle moving at a reasonable speed, but they have to push for a much longer time than the vehicle itself would take if powered by its own engine (which produces a larger force).

$$\text{Force} \times \text{time} = \text{impulse (Ns)}$$

Impulse is same as change in momentum.

$$\text{Force} = \frac{\text{change in momentum}}{\text{time}}$$

$$F = \frac{\Delta p}{\Delta t}$$

(Ns) (Ns)

Worked examples Pg 82, 83

- 1) A mass of 0.48 kg of water leaves a garden hose every second. The nozzle of the hose has a cross-sectional area of $8.4 \times 10^{-5} \text{ m}^2$. The water flows in the hose at a speed of 0.71 ms^{-1} . The density of water is 1000 kgm^{-3} .
- 2) An impulse of 85 Ns acts on a body of mass 5.0 kg that is initially at rest. Calculate the distance moved by the body in 2.0s after the impulse has been delivered.

Calculate

- a) the speed at which water leaves the hose
b) the force on the hose

Solution

$$a) \frac{\Delta m}{\Delta t} = \rho A v$$

$$\frac{0.48}{1} = 1000 \times (8.4 \times 10^{-5}) \times v$$

$$0.48 = 0.084 v$$

$$v = 5.71 \text{ ms}^{-1}$$

$$b) F = ma$$

$$F = 0.48 \left(\frac{v - u}{t} \right)$$

$$F = 0.48 \left(\frac{5.71 - 0.71}{1} \right)$$

$$= 0.48 \times 5$$

$$F = 2.4 \text{ N}$$

Change in momentum

$$\Delta p = F \Delta t = 85 \text{ kgms}^{-1}$$

$$85 = 5 \Delta v \quad \Delta p = m \Delta v$$

$$F = ma \quad 85 = 5 \Delta v$$

$$\Delta v = 17 \text{ ms}^{-1} \text{ is}$$

$$\text{change in velocity.}$$

$$v = 17 \text{ ms}^{-1} \text{ So, } 17 \times 2 = 34 \text{ ms}^{-1}$$

(17 x 2) because 17 is speed after impulse, then 2 seconds start.

- 3) An impulse I acts on an object of mass m initially at rest. Determine the kinetic energy gained by the object.

Solution Change in momentum = I

$$E_k = \frac{p^2}{2m}$$

$$E_k = \frac{I^2}{2m}$$

Mass = m

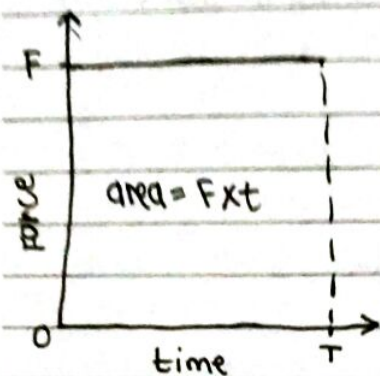
$$\text{OR } KE = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \left(\frac{I}{m} \right)^2 = \frac{I^2}{2m}$$

Force-time graphs

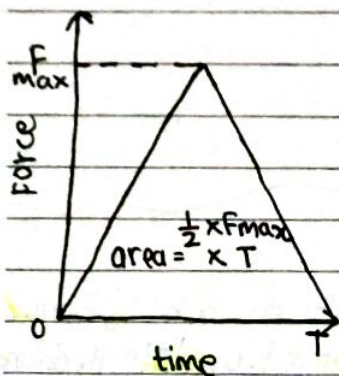
Until now we have assumed that forces are constant and do not change with time. But, this is rare. So, we need a way to cope with changes in momentum when the force is not constant.

$F = \frac{\Delta p}{\Delta t}$, suggests that we use a force-time graph.

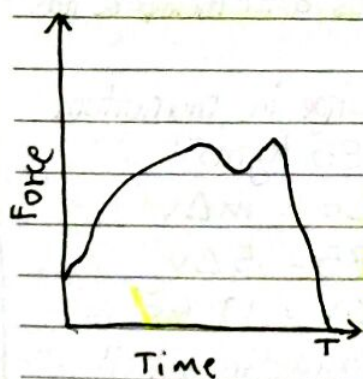


When the force is constant, the graph looks like this.

$\Delta p = F \times T$, ~~area~~ area below the line



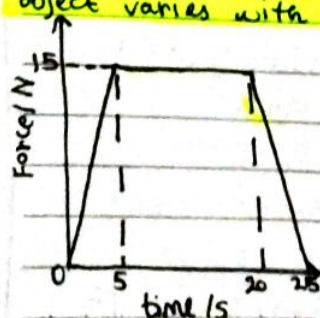
When force rises to the maximum (F_{max}) and falls back to zero in T . ~~Area~~ Area = $\frac{1}{2} \times F_{max} \times T = \frac{1}{2} F_{max} T$
 $\Delta p = \text{Area}$



When there is no obvious mathematical relationship between F and t , but there is still a graph. We have to estimate the number of squares and use area of one square to evaluate the momentum change.

Worked examples

- 1) The graph shows ^{how} the force acting on an object varies with time.



$$\begin{aligned} \Delta p &= \left(\frac{1}{2} \times 5 \times 15\right) + (15^2) + \left(\frac{1}{2} \times 5 \times 15\right) \\ &= 75 + 225 = 300 \text{ kg m s}^{-1} \\ &= 300 \text{ N s} \end{aligned}$$

$$\Delta p = m \Delta v$$

$$300 = 50v$$

$$\Delta v = 6 \text{ m s}^{-1}$$

$$0 + 6 = 6 \text{ m s}^{-1}$$

$$\text{Final speed} = 6 \text{ m s}^{-1}$$

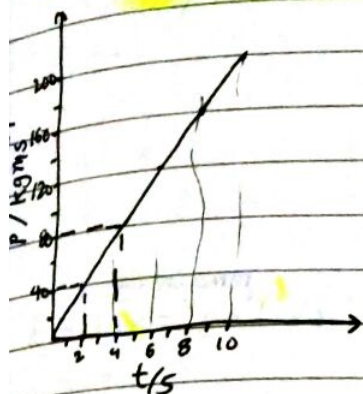
The mass of the object = 50 kg

initial speed = 0
 calculate time speed of object

2) The graph shows how the momentum of an object of mass 40 kg varies with time.

Calculate, for the object:

- a) the force acting on it
 b) the change in kinetic energy over the 10 s of the motion



$$\begin{aligned} \text{a) } \Delta p &= F \Delta t \\ 200 &= F(10) \\ F &= 20 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{b) } \Delta p &= 200 \\ \Delta E_k &= \frac{p^2}{2m} \\ &= \frac{200^2}{80} \\ &= \frac{40000}{80} \\ &= 500 \text{ J} \end{aligned}$$

Revisiting Newton's second law

We used $F = ma$ to show that $F = \frac{\Delta p}{\Delta t}$. Using full expression for p gives

$$F = \frac{\Delta(mv)}{\Delta t}$$

If differentiated using product rule:

$$\begin{aligned} F &= \frac{\Delta m}{\Delta t} v + \frac{\Delta v}{\Delta t} m \\ &= v \frac{\Delta m}{\Delta t} + m \frac{\Delta v}{\Delta t} \end{aligned}$$

This is usually 0, as mass keeps constant.

$$F = v \frac{\Delta m}{\Delta t} + m \frac{\Delta v}{\Delta t}$$

$m \frac{\Delta v}{\Delta t}$, simply means mass \times acceleration

$v \frac{\Delta m}{\Delta t}$, is new, means that instantaneous velocity \times $\frac{\text{change in mass}}{\text{change in time}}$

Worked Example (Example of theory on the next page)

A small fireworks rocket has a mass of 35 kg. The initial rate at which hot gas is lost from the firework has been lit at 3.5 g s^{-1} and the speed of release of this gas from the rear of the rocket is 130 m s^{-1} . Calculate the initial acceleration of the rocket.

$$\frac{\Delta m}{\Delta t} = 3.5 \frac{\text{g}}{\text{s}}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{v \Delta m}{m \Delta t}$$

$$v = 130 \text{ m s}^{-1}$$

$$= \frac{130 \times 3.5}{35} = 13 \text{ m s}^{-2}$$

$$\begin{aligned} F &= \frac{\Delta v}{\Delta t} m + \frac{\Delta m}{\Delta t} v \\ &= 0 + 0.0035(130) \\ &= 0.455 \text{ N} \end{aligned}$$

Rockets again

Rockets are excellent examples of momentum conservation in action. But, rockets are always losing mass. So m is not constant

$F = \frac{m\Delta v}{\Delta t} + \frac{v\Delta m}{\Delta t}$ is the new version of Newton's second law

and in this case, $F = 0$, because there are no external forces acting on the system.

$$\text{So, } \frac{m\Delta v}{\Delta t} = - \frac{v\Delta m}{\Delta t}$$

in $\frac{m\Delta v}{\Delta t}$, m is the instantaneous mass of rocket (including ^{remaining} fuel) and to the acceleration of this total mass $\frac{\Delta v}{\Delta t}$.

$\frac{v\Delta m}{\Delta t}$, refers to the ejection speed of the fuel and the rate at which

mass is lost from the system $\frac{\Delta m}{\Delta t}$. So, at one instant in time, the acceleration of the rocket

$$a = \frac{\Delta v}{\Delta t} = - \frac{v\Delta m}{m\Delta t}$$

- sign is because rocket is losing mass while gaining speed.

Helicopters

With the helicopter hovering, there is a weight force downwards, so

$$Mg = \frac{m\Delta v}{\Delta t} + \frac{v\Delta m}{\Delta t}, \text{ where } M \text{ is mass of helicopter}$$

Speed of helicopter = 0 because it is hovering. $\Delta v = 0$

Air gains momentum as it moves downwards. This is the speed, v , the air gains multiplied by the mass of the air accelerated every second so $Mg = v \frac{\Delta m}{\Delta t}$

weight of helicopter (in N) = mass of air pushed downwards per second (kg s^{-1})
 × speed of air downwards (m s^{-1})

Momentum and Safety

Seatbelts in cars are compulsory. There are also airbags to prevent passengers from striking the windscreen or the hard areas around it.

Physics of airbag and seatbelt

Someone in a car has to lose the same amount of kinetic energy and momentum whether they are stopped by windscreen or seatbelt. What differs is time during which the loss of energy and momentum occur. When no seatbelt, time to stop will be very short and the deceleration will therefore be very large. Large deceleration means large force \rightarrow higher damage.

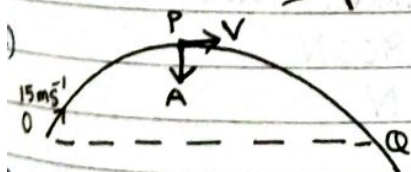
Seatbelts and airbags dramatically ^{increase} ~~reduce~~ the time taken by the occupants to stop and as $F \times \text{time} = \text{momentum change}$, for a constant change in momentum, a long stopping time implies a smaller, less damaging force.

Momentum and sport

At the start, we learnt that a table tennis ball will hurt less than a baseball. Now we know the reason for the difference. You should also realize that good technique in many sports hinges on the application of momentum change.

Many sports in which an object - usually a ball - is struck by hand, foot or bat rely on the efficient transfer of momentum. This transfer is often enhanced by a "follow through", which increases the contact time between the bat & ball. The player maintains the same force, but for a longer time, so impulse increases, increasing the momentum change as well.

Questions Pg 87



horizontal comp, $15 \cos 45$
 $= \frac{15}{\sqrt{2}} = 10.6 \text{ m s}^{-1}$

vertical comp, $15 \sin 45$
 $= 10.6 \text{ m s}^{-1}, \uparrow$

$u = 10.6$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$0 = 10.6^2 + 2(-9.81)(s)$$

$$-112.5 = 2(-9.81)(s)$$

$$s = 5.7 \text{ m}$$

$$25 + 5.7 = 30.7 \text{ m}$$

$$s = ut + \frac{1}{2}at^2$$

$$30.7 = \frac{1}{2}(9.81)t^2$$

$$t = 2.5 \text{ seconds}$$

$$v = u + at, v = 0 + (9.81)(2.5)$$

$$v = 24.6 \text{ m s}^{-1} = \text{vertical comp.}$$

$$\sqrt{10.6^2 + 24.6^2} = 26.7 \text{ m s}^{-1} = \text{speed}$$