Topic 6 Circular motion and gravitation

6.1 Circular motion

1) Essential idea - A force applied perpendicular to a body's displacement can result in its circular motion.

2) The force must act inwards in all cases of circular motion.

3) Data booklet

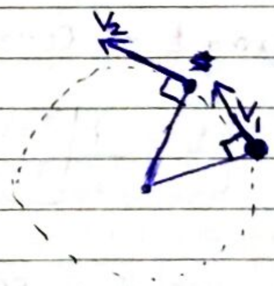
1. $v = \omega r$

2. $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$

3. $F = \frac{mv^2}{r} = m\omega^2 r$

4) Utilization - Motion of charged particles in magnetic fields .Unit 5.4

5) A particle is said to be in uniform circular motion if it travels in a circle (or arc) with constant speed **v**.



Magnitude of velocity does not change.
Direction of velocity does change.
Thus, there is an acceleration although speed does not change.

$a = \frac{\Delta v}{\Delta t}$, $\Delta v = v_2 - v_1$

$\Delta v = v_2 + (-v_1)$

Acceleration is towards the center.
Centripetal = center seeking

~~$F_c = ma_c$~~

6) $\boxed{F_c = ma_c}$ centripetal force

↓ radius = ↑ F_c

↑ velocity = ↑ F_c

Date

7) $\boxed{a_c = \frac{v^2}{r}}$ centripetal acceleration

E.g. 730 kg car negotiates 30 m radius turn at $25 ms^{-1}$. Centripetal acceleration and force? What force is causing this acceleration?

$$a_c = \frac{v^2}{r} = \frac{25^2}{30} = \frac{625}{30} = 20.8\ ms^{-2} = 21\ ms^{-2}$$

$F_c = ma_c$

$F_c = 730 \times 20.8 = 15208\ N$

$= 1.5 \times 10^4\ N$

The centripetal force is caused by the friction between the tires and the pavement.

8) Period T - Period T is the time for one complete revolution.

Frequency f - Cycles or oscillation or revolutions per second. Hz

$\boxed{f = \frac{1}{T}}$ Period and Frequency

Find period & frequency of one day.

Period $T = 24 \times 60 \times 60 = 86400 s$

Frequency $f = \frac{1}{86400} = 1.16 \times 10^{-5}\ Hz$

9) Velocity ~~of~~ and period

One revolution is circumference $C = 2\pi r$

Therefore $v = \frac{distance}{time} = \frac{2\pi r}{T}$

$$v = \frac{2\pi r}{T}$$

$$v^2 = \frac{4\pi^2 r^2}{T^2}$$

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r^2}{T^2 r} = \frac{4\pi^2 r}{T^2}$$

Date

$a_c = \frac{v^2}{r}$		Centripetal acceleration
$a_c = \frac{4\pi^2 r}{T^2}$		

Example

2.50 kg cat swung by string having radius 3.00 meters. He takes 5.00 seconds to complete one revolution. What are a_c and F_c?

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$a_c = \frac{4\pi^2(3)^2}{5^2 \times 3}$$

$$a_c = \frac{36\pi^2}{75}$$

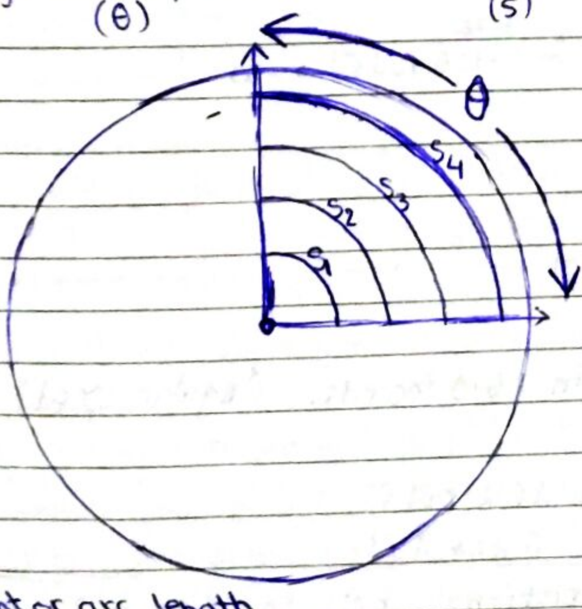
$$a_c = 4.74\ ms^{-2}$$

$$F_c = ma_c$$

$$F_c = 2.50 \times 4.74$$

$$F_c = 11.8\ N$$

10) Angular displacement (θ) and arc length (s)



Radians & Degrees
π rad = 180° = $\frac{1}{2}$ rev.
2π rad = 360° = 1 rev
Radian-degree-revolution conversions

displacement or arc length or distance

$S = r\theta$	θ in radians	relation between s and θ

radius Angular displacement

$\omega = \frac{\theta}{t}$	Angular velocity or angular speed	$rad\,s^{-1}$

$$v = \frac{s}{t} = \frac{r\theta}{t} = r\left(\frac{\theta}{t}\right) = r\omega = v$$

$v = r\omega$	relation between v and ω

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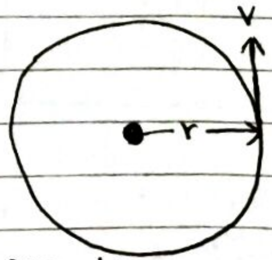
Example

Q Mass moving at constant speed v in a circle of radius r as shown.



Find...

a) the period T of the point mass, and

b) the Frequency F of the point mass, and

c) the angular speed ω of the point mass.

~~T = 12 seconds~~

a) $v = \frac{s}{t} = \frac{2\pi r}{T}$

$T = \frac{2\pi r}{v}$

b) $F = \frac{1}{T} = \frac{v}{2\pi r}$

c) $\omega = \frac{v}{r}$ ~~= 2πr~~

Q) Find the angular speed of the second hand on a clock. Find speed of the tip of the hand if it is 18.0 cm long.

$\omega = \frac{\theta}{t} = \frac{2\pi}{60} = \frac{\pi}{30} = 0.105 \text{ rad s}^{-1} = \omega$

$v = r\omega$

$v = 18 \times 10^{-2} \times 0.105$

~~v = 0.019~~ $v = 0.0188 \text{ ms}^{-1}$

Q) Car rounds a 90° turn in 6.0 seconds. Angular speed?

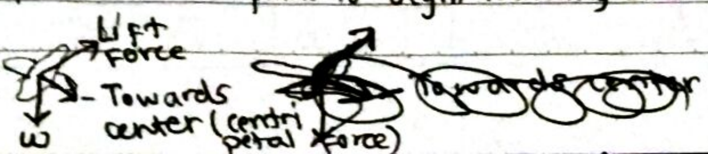
$\omega = \frac{\frac{\pi}{2}}{6} = \frac{\pi}{12} = 0.262 \text{ rad s}^{-1}$

$= 0.26 \text{ rad s}^{-1}$

The car can turn due to the friction between tire and pavement.
The friction always points towards center of circle.

This is banking. (Considered qualitatively only in syllabus)

- As a plane banks (rolls), lift vector begins to have a horizontal component.
- The centripetal force causes the plane to begin traveling in a horizontal circle.



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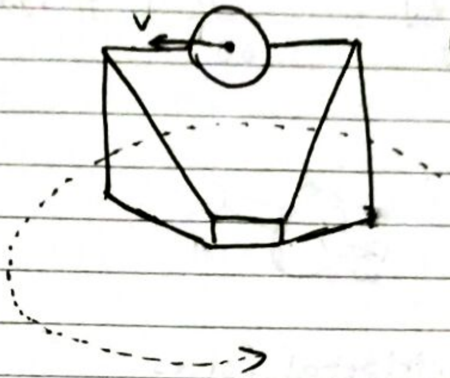
Even though cars use friction, roads are **banked** so that the need for friction is reduced.

A component of the normal force provides centripetal force.



(like an uplifted curve or corner) (Maut ka Kua!)

(It can turn even if it is frictionless.)

11) Angular speed and centripetal acceleration

$v = r\omega$

$a_c = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2$

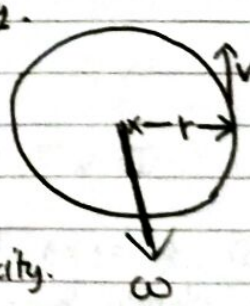
$a_c = r\omega^2$ $a_c = \frac{4\pi^2 r}{T^2}$ $a_c = \frac{v^2}{r}$	$F_c = m\omega^2 r$ $F_c = \frac{4\pi^2 mr}{T^2}$ $F_c = \frac{mv^2}{r}$	a_c and F_c (all three forms)

$\omega = \frac{2\pi}{T} = 2\pi f = \frac{\theta}{t}$	relation between θ, T and f

Angular speed with a direction is called angular velocity.

Direction of a rotation. Right hand rule:

1) Rest heel of your right hand on the rotating object.
2) Make sure your fingers are curled in the direction of rotation.
3) Your extended thumb points in the direction of the angular velocity.



$\boxed{\text{Angular velocity } (\omega) \text{ always points perpendicular to the plane of motion.}}$

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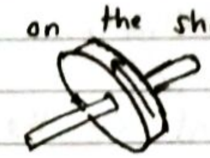
Example

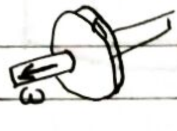
Find angular velocity (in $rad\,s^{-1}$) of the wheel on the shaft. It is rotating at 30.0 rpm (revolutions per minute).



$$\omega = \frac{\theta}{t} = \frac{30 \times 2\pi}{60} = \frac{60\,\pi}{60} = \pi \ rad\,s^{-1}$$

Magnitude = $3.14\ rad\,s^{-1}$

Direction = RHR =



12) Forces providing centripetal forces

Example

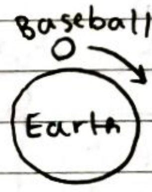
Friction (Race car making a turn)

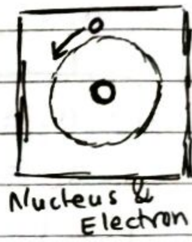
Tension (Albert the physics cat spinned)

Gravitational Force (Baseball and the Earth)

Electric force (an electron orbiting a nucleus)

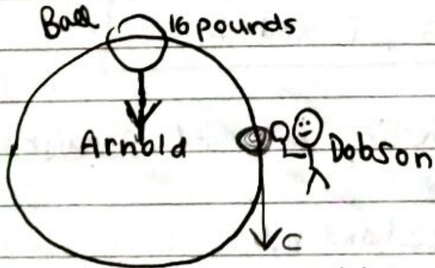
Magnetic force (a moving charge in a B-field)





Example

1) Dobson watching 16-pound bowling ball being swung around at $50\ ms^{-1}$ by Arnold. If string is cut at the instant the ball is next to the ice cream, what will the ball do?



It will fly tangent to the original circular path along C.

2) 0.500 kg Baseball is placed in a circular orbit around earth at slightly higher than tallest point 8850 m, mount everest. Given $R_E = 6400000\,m$ is radius of Earth, Find the speed of ball.

~~ΣF = 0 N~~ F_c is caused by gravitational force.

$mg = 0.500 \times 9.81 = 4.905\ N$

~~4.905~~ $mg = \frac{mv^2}{r}$

$$\sqrt{\frac{4.905 \times 6408850}{0.5}} = v$$

$v = 8000\ ms^{-1}$

~~$\sqrt{\frac{4.905 \times 6400000}{0.500}} = v = 8000\ ms^{-1}$~~

b) How long will it take the ball to return to everest?

$$v = \frac{ds}{t}$$

$$8000 = \frac{2\pi(6408850)}{T}$$

$$T = 5033.5 \text{ seconds}$$
$$= 1.410 \text{ hours}$$

c) Explain how an object can remain in orbit yet always be falling?

When the ball is thrown at a small speed, the ball is drawn by gravity towards the center of the earth. But, when the ball is finally thrown at a great enough speed that it matches the curvature of the Earth, the ball is effectively falling around the Earth while in orbit.

Q) Angular speed of the minute hand of a clock?

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = \frac{2\pi}{3600}$$

$$= 0.00175 \text{ rad s}^{-1}$$

~~Angular~~ Angular speed of ~~speed~~ Rotation of the Earth in one day?

~~$r = 6.4 \times 10^6$ m~~

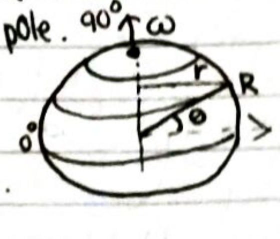
~~$2\pi r = 2\pi(6.4\times10^6) = 4.02\times10^7$ m~~

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{60\times60\times24} = 7.27\times10^{-5} \text{ rad s}^{-1}$$

The small angular speed is why we can't feel the Earth as it spins.

Example

Find the apparent weight of someone standing on an equatorial scale if his weight is 882 N at the North pole.



$\omega = 0.0000727 \text{ rad s}^{-1}$

Lines are line of a latitude.

R line is earth's radius

r represents the radius of a circle a point at a latitude of θ follows.

$r = R\cos\theta$

$\theta =$ 90° at the pole.
0° at equator.

$\overset{R}{\xleftarrow{W}\longrightarrow}$ $\overleftarrow{a_c}$

So, at equator $r = R$, and at the pole, $r = 0$. $R = 6400000$ m.

At the equator,

$$a_c = r\omega^2 = 6.4\times10^6 \times 0.0000727^2 = 0.0338 \text{ ms}^{-2}$$

At the pole,

$$a_c = r\omega^2 = 0 \times 0.0000727^2 = 0 \text{ ms}^{-2}.$$

$$\Sigma F = ma$$
$$W - R = F_c$$
$$W = F_c + R$$
$$R = W - ma_c = 882 - \left(\frac{882}{9.8} \times 0.0338\right) = 879\text{N}$$

Man lost 3N.

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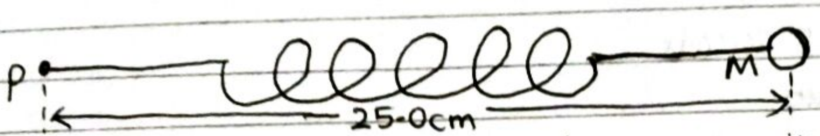
Example

A linear spring of negligible mass requires 18.0N to cause its length to increase by 1.0 cm. A sphere of mass 75.0g is attached to one of the spring. Distance between centre of ~~spring~~ sphere M & other end P of the unstretched spring is 25.0 cm, as shown below



Sphere is rotated at constant speed in horizontal circle with centre P. PM increases to 26.5cm

a) Explain why the spring increases in length when the sphere is moving in a circle.

$F = kx = \frac{mv^2}{r}$. As velocity increases, force increases to keep it moving in a circle. In this case, x and v^2 are ~~proportional~~ directly proportional. ~~to force~~

So, x increases.

b) Determine the speed of the sphere.

~~$F = 18 \times 0.015 = 0.27\,N$~~

~~$0.27 = \frac{0.075 \times v^2}{0.25}$~~ ~~$v = 0.95\,ms^{-1}$~~

~~$\frac{0.27 \times 0.25}{0.075} = v^2$~~

Solution

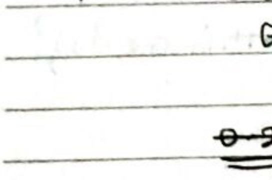
$18.0\,N\,cm^{-1} = 1800\,N\,m^{-1}$, $k = 1800\,N\,m^{-1}$

$F_c = kx$, $F_c = 1800 \times 0.015 = 27\,N$

$27 = \frac{0.075 \times v^2}{0.265}$

$\sqrt{\frac{27 \times 0.265}{0.075}} = v = 9.77\,ms^{-1}$

Example



Point P has linear speed v and centripetal acceleration a. What about point Q?

$v_1 = R\omega$ $v_2 = 2R\omega$

$2v_1 : v_2$

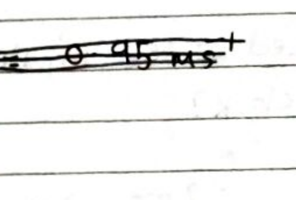
$a_1 = R\omega^2$ $a_2 = 2R\omega^2$

$2a_1 : a_2$

Linear speed : 2v

Cent. Acceleration : 2a

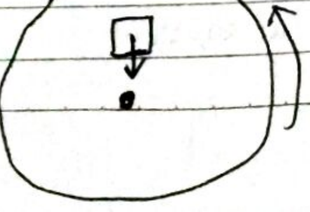
Q



Flat horizontal disc

Rotating at constant speed. Draw the horizontal force or forces acting on the brick?

Centripetal acceleration is always towards center. (center-seeking)



Topic 6 Circular motion and gravitation

Date

6.2 Newton's law of gravitation

1) Newton's Law of Gravitation

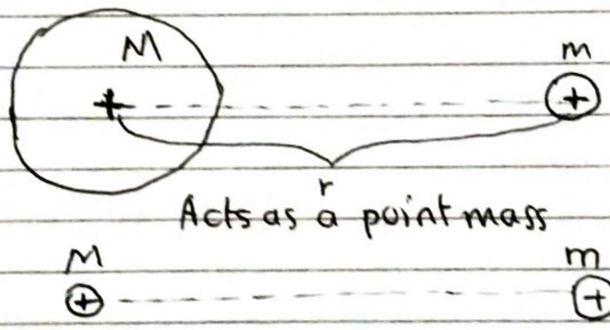
- The gravitation force is the weakest of all the 4 fundamental forces.

- Newton's law of gravitation - gravitational force between two point masses m_1 and m_2 is proportional to their product, & inversely proportional to the square of their separation r.

$F = \frac{Gm_1m_2}{r^2}$ where $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$	Universal law of gravitation

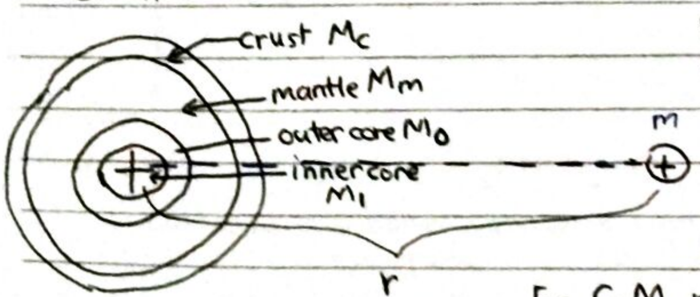
G is universal gravitational constant. Discovered by Henry Cavendish

The celestial bodies we observe are point masses. They have layers. "A spherically symmetric shell of mass M acts as if all of its mass is located at its center." - Newton



2) Solving problems involving gravitational force

Earth



Assuming each shell is symmetric, the gravitational force caused by that shell acts as though its mass is all concentrated at its center.

Net force at m:

$$F = \frac{GM_1m}{r^2} + \frac{GM_om}{r^2} + \frac{GM_mm}{r^2} + \frac{GM_cm}{r^2}$$

$$F = \frac{G(M_1 + M_o + M_m + M_c)m}{r^2}$$

Thus, $F = \frac{GMm}{r^2}$ where $M = M_1 + M_o + M_m + M_c$

M is just total mass of Earth lah!

Date

r is distance between the centers of the masses.

Example

Earth has a mass of $M = 5.98 \times 10^{24}$ kg and moon has mass of $m = 7.36 \times 10^{22}$ kg. Mean distance between Earth & moon is 3.82×10^{8} m. What is the gravitational force between them?

$$F = \frac{GMm}{r^2} = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 7.36 \times 10^{22}}{(3.82 \times 10^{8})^2}$$

$$\cancel{= 7.68 \times 10^{28}\ N}$$

$$= 2.01 \times 10^{20}\ N$$

Radius of each mass is immaterial here because we are already referring to point masses. So, don't worry about adding it here.

Example

Moon mass 7.36×10^{22} kg. Mean distance between earth & mooo is 3.82×10^{8} m.
What is the speed v of moon in its orbit about Earth?

$$F_G = 2.01 \times 10^{20}\ N$$

$$F_G = F_c$$

$$2.01 \times 10^{20} = \frac{7.36 \times 10^{22} \times v^2}{3.82 \times 10^{8}}$$

$$\frac{2.01 \times 10^{20} \times 3.82 \times 10^{8}}{7.36 \times 10^{22}} = v^2$$

$$v = 1021.4\ ms^{-1}$$

$$v = 1020\ ms^{-1}$$

$$v = 1.02 \times 10^{3}\ ms^{-1}$$

For circular orbits, the F_G is the F_c.

$$F_G = F_c$$

b) what is period of moon (in days) in its orbit around Earth?

One cycle time

$$\frac{d}{v} = t$$

$$t = \frac{2\pi \times 3.82 \times 10^{8}}{1.02 \times 10^{3}} = 2353114.5 \text{ seconds}$$

$$\frac{2353114.5}{86400} = 27.2 \text{ days}$$

3) Gravitational Field Strength

m is located at distance r from M.

Gravitational field strength g is force per unit mass on m due to presence of M.

$g = \frac{F}{m}$	Gravitational Field strength	$N kg^{-1}$

$F = ma$

$N = kg \times ms^{-2}$

$\frac{N}{kg} = ms^{-2}$

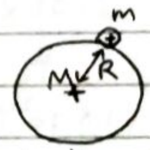
$\boxed{N kg^{-1} = ms^{-2}}$

$F = mg$ — On Earth's surface $g = 9.8\, ms^{-2}$

g is none other than the gravitational field strength.

4) Solving problems involving gravitational field strength

Suppose m is on surface of M with radius R.



From universal law of gravitation, weight of m is equal to its attraction to the planet's mass M and equals $F = \frac{GmM}{R^2}$

$$mg = \frac{GMm}{R^2}$$

$g = \frac{GM}{R^2}$	Gravitational field strength at surface of a planet of mass M and radius R.

The same derivation works for any r.

$g = \frac{GM}{r^2}$	Gravitational field strength at distance r from center of a plant of mass M

Date

Practice: Mass of Earth is $M = 5.98 \times 10^{24}$ kg and the radius of Earth $R = 6.37 \times 10^6$ m. Find gravitational field strength at Earth's surface, and at a distance of one Earth radius ~~surface~~ above its surface.

$$g = \frac{GM}{r^2}$$

$$g_1 = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6.37 \times 10^6)^2} = 9.83 \ \text{N kg}^{-1} \ (\text{m s}^{-2})$$

$$g_2 = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6.37 \times 10^6 \times 2)^2} = 2.46 \ \text{N kg}^{-1} \ (\text{m s}^{-2})$$

Practice: Satellite 525 kg is launched from Earth's surface to a height of one earth radius above the surface. a) Weight at surface and b) at altitude?

$$g = \frac{GM}{r^2}$$

$$g_1 = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6.37 \times 10^6)^2} = 9.83 \ \text{N kg}^{-1} \ (\text{m s}^{-2})$$

$$W = mg_1 = 525 \times 9.83 = 5.16 \times 10^3 \ \text{N}$$

$$g_2 = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(2 \times 6.37 \times 10^6)^2} = 2.46 \ \text{N kg}^{-1} (\text{m s}^{-2})$$

$$W = mg_2 = 525 \times 2.46 = 1290.2 \ \text{N}$$
$$= 1.29 \times 10^3 \ \text{N}$$

5) Gravitational Field strength (again)

Compare: $F = \frac{GMm}{r^2}$ $\qquad$ $g = \frac{GM}{r^2}$

(Force - action at a distance) $\qquad$ (Field - local curvature of space)

(Two masses, and force is the result of their interaction at a distance r.) $\qquad$ (Just one mass, that sets up the local field in the space surrounding it. It "curves" it.)

Field view of universe is preferred over the force view ~~of~~ action at a distance. "Force signal" will be slightly delayed in telling orbital mass when to turn. End result would be an expanding spiral motion. We do not observe plants leaving their orbits as they travel around the sun. Thus, action at a distance doesn't work if we believe special relativity. Current evidence points to correctness of special relativity. Field view eliminates the need for long distance signaling between 2 masses. It distorts space about one mass.

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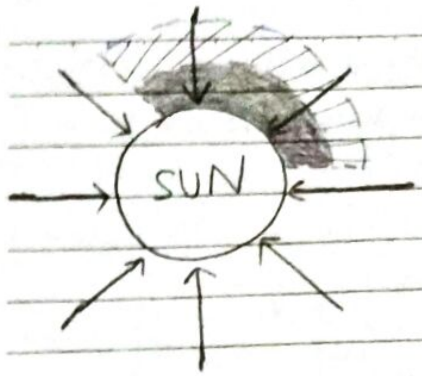
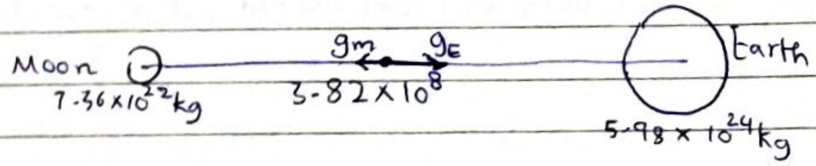


- We look at concentration of field lines to know how strong the field is at a particular point in the vicinity of a mass.
- The closer together the field lines, the ~~closer~~ stronger the field.
- Shaded portion lines are closer together than striped portion.
- Shaded field is thus stronger than striped field.

6) Solving problems involving gravitational field strength - (again)

Q Find gravitational field strength at a point between the Earth and the moon that is right between their centers.



$$r = \frac{3.82 \times 10^8}{2} = 1.91 \times 10^8 \, m$$

$$E_m = \frac{6.67 \times 10^{-11} \times 7.36 \times 10^{22}}{(1.91 \times 10^8)^2} \qquad E_E = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(1.91 \times 10^8)^2}$$

$$E_m = 0.000135 \, N \, kg^{-1} \qquad E_E = 0.0109 \, N \, kg^{-1}$$

$0.0109 - 0.000135 = 0.010765$ N, ~~rightwards~~ towards the Earth.

$= 1.08 \times 10^{-2} \, N kg^{-1}$

Practice

Jupiter's gravitational field strength at its surface is $25 \, N kg^{-1}$ while its radius is 7.1×10^7 m. a) Deduce jupiter's mass. b) Weight of 65 kg man on Jupiter.

a) $$25 = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times M}{(7.1 \times 10^7)^2}$$

$$\frac{25 \times (7.1 \times 10^7)^2}{6.67 \times 10^{-11}} = M = 1.89 \times 10^{27} \, kg$$

b) $25 \times 65 = 1625 \, N$

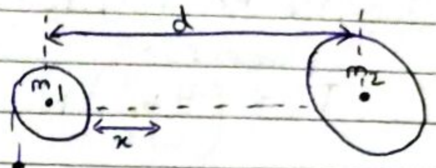
$= 1600 \, N$

KIKY

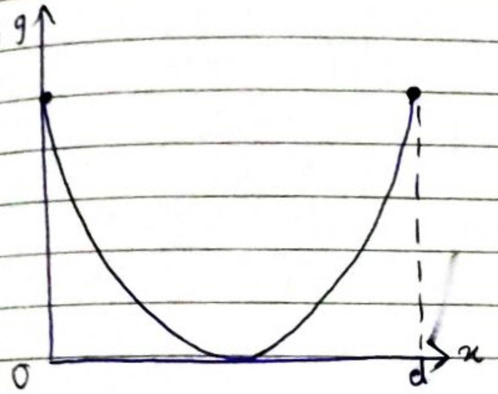
Date

Practice : Two spheres of equal mass & different radii are held a distance d apart. The gravitational field strength is measured on the line joining the 2 masses at position x which varies.

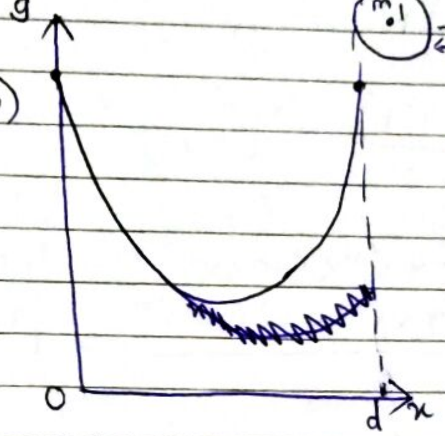
Which graphs shows the variation of g with x correctly?



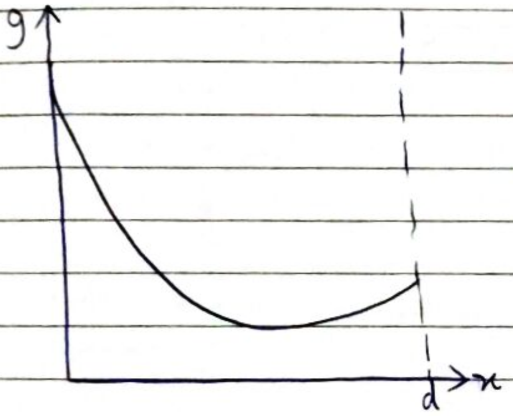
a)



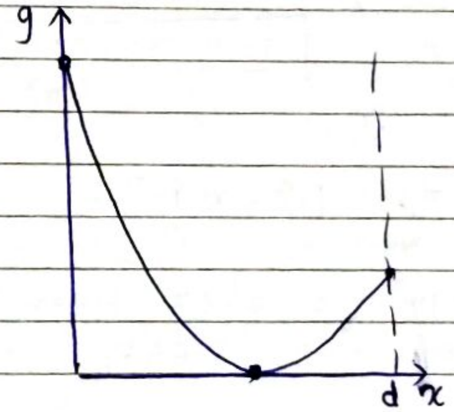
b)



c)



d)



At the surfaces, both masses have their ~~big~~ strongest E. But, m_2 has higher radius, so its ~~maximum~~ E has surface is lower than M_1. a & b are wrong.

Somewhere closer to m_2, $r_1 + x = r_2 + (d - x)$, so E is 0. C is wrong.
$\boxed{D}$ is correct.

) Solving problems involving orbital ~~speed~~ period.

Kepler's law - period T of an object in a circular orbit about a body of mass M is given by $T^2 = [(4\pi^2/(GM)]r^3$. Derive it.

$F_c = ma_c$, in circular orbit.

$F_c = \frac{GMm}{r^2}$, from newton's law of gravitation

$a_c = \frac{4\pi^2 r}{T^2}$, from 6.1

$\cancel{m}a_c = \frac{GM\cancel{m}}{r^2}$

$\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}$

$4\pi^2 r^3 = GMT^2$

$T^2 = \frac{4\pi^2 r^3}{GM}$

$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$

(Not in data booklet!) Memorize!

Example A satellite in geosynchronous orbit takes 24 hours to orbit the Earth. So, it can be above same point of Earth at all times, if desired. Find the necessary radius & express it in terms of earth radii. $R_E = 6.37 \times 10^6$ m.

Kepler's law

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$86400^2 = \frac{4\pi^2 r^3}{GM}$$

$$86400^2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24} = 4\pi^2 r^3$$

$$\frac{86400^2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{4\pi^2} = r^3$$

$$r = \sqrt[3]{\frac{86400^2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{4\pi^2}}$$

$$= 4.23 \times 10^7 \text{ m}$$

$$\frac{4.23 \times 10^7}{6.37 \times 10^6} = 6.63 \text{ times Earth's radius.}$$

$$= 6.63 R_E$$

Kepler Third Law, Not in data booklet	$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$	$T \propto r^3$ $\frac{4\pi^2}{GM}$ is constant

Initially, this was just $T^2 \propto r^3$. Newton's law was needed to get the proportionality constant.

Example - Rings of Saturn are made of rocky particles that orbit the planet. Period T of each particle depends on distance r from the centre of Saturn. T is proportional to R^n.

what is n?

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$\sqrt{T^2} = \sqrt{\frac{4\pi^2}{GM}(r^3)}$$

$$T = \left(\frac{4\pi^2}{GM}\right)^{\frac{1}{2}} r^{\frac{3}{2}}$$

$$n = \frac{3}{2} = 1.5, \quad T \propto r^{1.5}$$

Date

Q) Two satellites X & Y, move in circular orbits about Earth. Orbital period of satellite X is eight times of satellite Y.

Find ratio $\frac{\text{orbital radius of satellite X}}{\text{orbital radius of satellite Y}}$?

$\cancel{T^2 = \left(\frac{4\pi^2}{GM}\right) r^3}$

$\cancel{y \quad x}$

$\cancel{T_y^2 : (8T_y)^2}$

$\cancel{T_y^2 : 64T_y^2}$

~~64 times period of y when squared~~

$\cancel{T^2 \propto r^3}$

$\cancel{T^2 = kr^3}$

$\cancel{64T^2 = kr^3 \times 64}$

$\cancel{r : 4r}$

$\boxed{\text{4 is the ratio.}}$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$T_x^2 = \frac{4\pi^2}{GM} r_x^3 \qquad T_y^2 = \frac{4\pi^2}{GM} r_y^3$$

$$T_x = 8T_y$$

$$T_x^2 = 64T_y^2$$

$$\frac{T_x^2}{T_y^2} = 64$$

$$1 = \frac{\cancel{\frac{4\pi^2}{GM}} r_x^3}{\cancel{\frac{4\pi^2}{GM}} r_y^3}$$

$$64 = \frac{r_x^3}{r_y^3} = \frac{T_x^2}{T_y^2}$$

$$(64)^{\frac{1}{3}} = \left(\frac{r_x^3}{r_y^3}\right)^{\frac{1}{3}}$$

$$\boxed{4 = \frac{r_x}{r_y}}$$

8) Solving problems involving gravitational field.

If Dobson ~~m~~ is in an elevator, and he drops a ball - Accelerate downward is 10 ms^{-2}.

Q But, what about when ~~the doctor~~ elevator is accelerating upwards at 2 ms^{-2}? What is a?

Since, elevator moves 2 ms^{-2} upwards to meet the ball, and ball goes down at 10 ms^{-2}, Dobson would observe an acceleration of 12 ms^{-2}.

If elevator was going down at 2 ms^{-2}, then he will observe ball to be accelerating at 8 ms^{-2}.

If elevator goes at 10 ms^{-2} down, 0 acceleration observed for ball. He thinks it is "weightless".

Date

Astronauts feel weightlessness because, they and the spacecraft, both accelerate at the same value. $a_c = g$. They fall together and appear weightless.

In deep space, we are truly weightless.
r is so large for every m in $\frac{GMm}{r^2}$, that Force (F), the force of gravity, is almost zero.

Practice

Satellite of mass m and speed v orbits the Earth at distance r from centre of the Earth. The gravitational field strength due to earth is ?

$\cancel{F_c = \frac{mv^2}{r} = \frac{GMm}{r^2}}$

$\cancel{F_c = v^2 = \frac{GM}{r}}$

$\cancel{E = \frac{GM}{r^2}}$

$$F = mg = \frac{mv^2}{r} = \frac{GMm}{r^2}$$

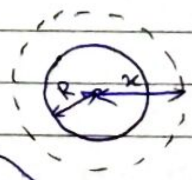
Cancel m

$$E = g = \frac{v^2}{r} = \frac{GM}{r^2}$$

~~Acceleration~~

Gravitational field strength is $\frac{v^2}{r}$.

Practice A satellite of mass m orbits a planet of mass M and radius R as shown. Radius of circular orbit is x. The planet can be as point mass with mass concentrated at center.



a) Deduce that linear speed v of satellite in its orbit is $v = \sqrt{\frac{GM}{x}}$.

$$F_c = \frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$= \frac{mv^2}{\cancel{R+x-R}} = \frac{GMm}{x^2}$$

$$\frac{\cancel{m}v^2}{x} = \frac{GM\cancel{m}}{x^2}$$

$$v^2 = \frac{GM}{x}$$

$$v = \sqrt{\frac{GM}{x}}$$

b) Derive expression in m, G, M and x, for kinetic energy of satellite.

$$K_E = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m \times \left(\frac{GM}{x}\right)$$

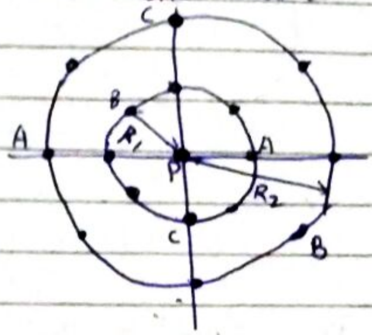
$$= \frac{1}{2} \times \frac{GMm}{x} = \frac{GMm}{2x}$$

Date

4) Solving problems with gravitational force

Practice A binary star consists of 2 stars that each follow circular orbits about a fixed point.

They have same orbital period T. Can be considered as point masses with mass concentrated at its centre. The stars, masses M_1 and M_2 orbit at distances R_1 & R_2 respectively from point P.



a) Force that provides the centripetal force?

Gravitational Force

b) By considering the force acting on one of the stars, deduce orbital period T is given by $T^2 = \frac{4\pi^2}{GM_2} R_1 (R_1 + R_2)^2$.

~~$F_1 = \frac{GMm_1}{r_1^2}$ $F_2 = \frac{GMm_2}{r_2^2}$~~

$$F = \frac{GM_1M_2}{(R_1+R_2)^2}$$

They always have radius $R_1 + R_2$ between them.

$$F = \frac{GM_1M_2}{(R_1+R_2)^2} = \frac{mv^2}{r}$$

$$v = \frac{2\pi r_1}{T}$$

$$\frac{mv^2}{r_1} = \frac{m_1 \times 4\pi^2 r^2}{r_1 T^2}$$

$$\frac{G\cancel{M_1}M_2}{(R_1+R_2)^2} = \frac{\cancel{m_1} \times 4\pi^2 r_1^2}{r_1 T^2}$$

$$M_2 G T^2 = (4\pi^2 R_1) \times (R_1+R_2)^2$$

$$T^2 = \frac{4\pi^2 R_1 (R_1+R_2)^2}{GM_2}$$

~~$T^2 = \frac{4\pi^2}{GM} r^3$~~

~~$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$~~

~~$T_1 = T_2$~~

~~$T = \frac{1}{f}$ $v = \frac{2\pi r}{T}$~~

~~$\frac{GMm_2}{r^2} = \frac{mv^2}{r}$~~

~~$\frac{GMm_2}{r^2} = \frac{m_2 \times \frac{4\pi^2 r^2}{T^2}}{r_2}$~~

~~$\frac{GMm_2}{r_2^2} = \frac{4\pi^2 m_2 r^2}{r_2 T^2}$~~

~~$\frac{GM}{r_2} = \frac{4\pi^2 r_1^2}{T^2}$~~

~~$T^2 GM = 4\pi^2 r^3$~~

~~$T^2 = \frac{4\pi^2 r^3}{GM}$~~

$r_2 = r_1 + (r_2 - r_1)$

c) M_1 is closer to point P than M_2. Using (b), state & explain which star has the larger mass.

$$T^2 = \frac{4\pi^2 R_1 \cancel{(R_1+R_2)^2}}{GM_2} = \frac{4\pi^2 R_2 \cancel{(R_2+R_1)^2}}{GM_1}$$ (Same period)

~~$T \propto \frac{R_1 (R_1+R_2)^2}{M_2}$~~

~~T is same.~~

~~$M_2 \propto R_1 (R_1+R_2)^2$~~

$$\frac{R_1}{M_2} = \frac{R_2}{M_1}$$

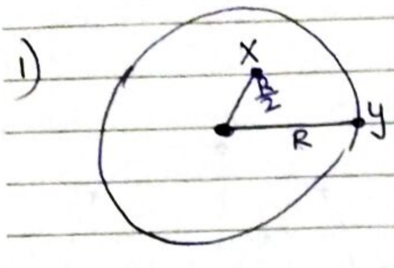
$$R_1 M_1 = R_2 M_2$$

M_1 has larger mass than M_2 because $R_2 > R_1$.

KKY

Topic 6 Self-test Questions Cambridge Date

1)



X, $a_c = a$

$V = V$

$v = \omega r$

$\boxed{V = \omega \times \frac{R}{2}}$

$a = \frac{v^2}{r} = \omega^2 r$

$\boxed{a = \omega^2 \frac{R}{2}}$

y, $r = R$

$v = \omega r$

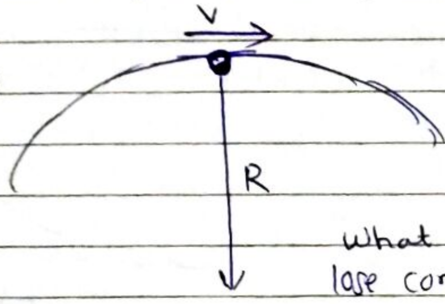
$\boxed{v = \omega R}$

$a = \omega^2 r$

$\boxed{a = \omega^2 R}$

D. Speed $= 2v$, Acceleration $= 2a$

2)



Car moves along a path that is part of a vertical ~~radius~~ circle of radius R. Speed of car at highest point is V.

What is the max value of V so that the car does not lose contact with road?

$\frac{mv^2}{r} = mg - N$ (normal force)

$\frac{mv^2}{r} = mg - N$

$mg - \frac{mv^2}{r} = N$

If, car loses contact, ~~mg = N~~ ~~N = mg~~ Centripetal force is not enough to keep it on track. It is when $mg = N$, and resultant vertical force is 0.

~~$mg - \frac{mv^2}{r} = mg$~~

$\frac{mv^2}{r} = mg$

$v^2 = rg$

$v = \sqrt{Rg}$

B. $v_{max} = \sqrt{gR}$

(3) Particle moves along horizontal circle with constant speed. Magnitude & direction of current?

$F = \frac{kq_1q_2}{r^2}$

$a = \frac{kq_1}{r^2} = \frac{v^2}{r}$

$\frac{ar^2}{k} = q_1$

	Mag.	Direc.
A	Constant	Changing
B	Constant	Constant
C	Changing	Changing
D	Changing	Constant

a is changing in direction. Magnitude is 0. So, magnitude of q ~~is 0 or~~ constant. Direction changing.

KIKY

Date

4) A probe orbits a planet in circular orbit of radius r. Period T. What is the mass of the planet?

(a) $\frac{4\pi^2 r^3}{GT^2}$

b) $\frac{4\pi^2 r}{GT^2}$

c) $\frac{4G\pi^2 r^3}{T^2}$

d) $\frac{4G\pi^2 r}{T^2}$

$T = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{T}$

$\cancel{F = \frac{Gm_1 m_2}{r^2}}$

$F = ma = \frac{mv^2}{r}$

$v = \omega r$ $\quad F = m\omega^2 r$

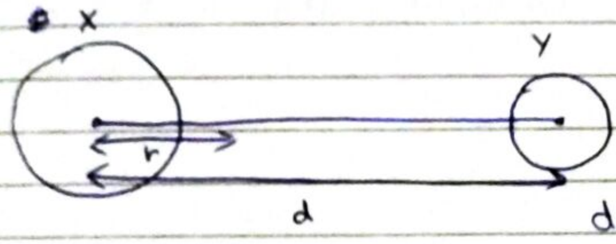
$\frac{GM\cancel{m}}{r^2} = \cancel{m}\omega^2 r$

$\omega^2 r = \frac{4\pi^2 r}{T^2}$

$\frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}$

$GM = \frac{4\pi^2 r^3}{T^2}$, $\quad M = \frac{4\pi^2 r^3}{T^2 G}$, Option A.

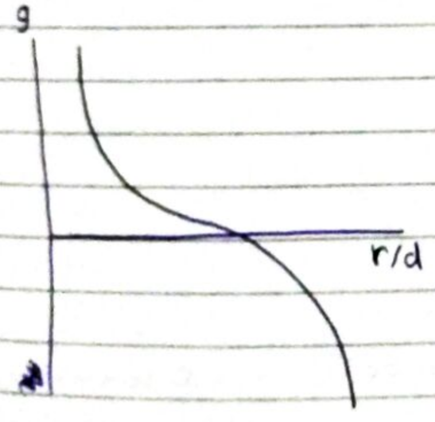
5)



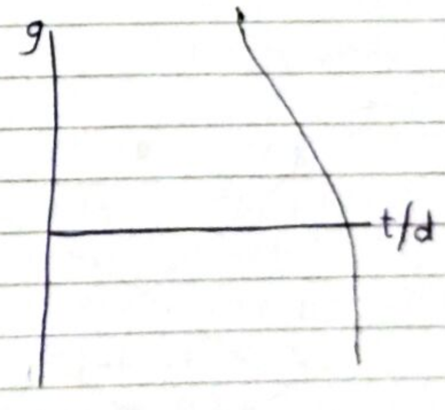
Mass of X is 4 times mass of Y.
Net gravitational field strength along the dotted line a distance r from centre of X is g. Positive g means field is towards right.

Which graph shows the variation g with r/d ?

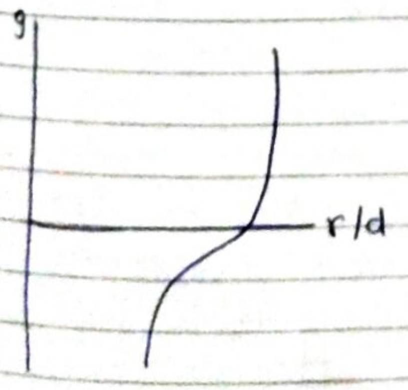
A



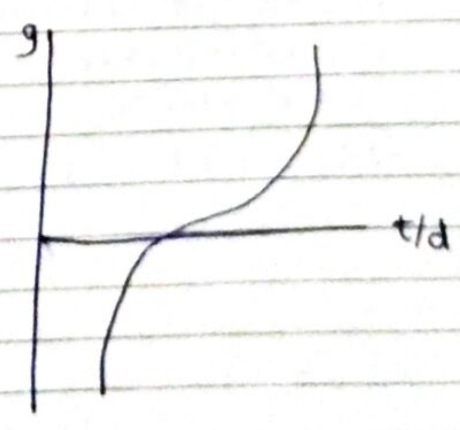
B



C



D



Solved on next page.

(KKY)

Date

As r/d increases, r increases

$m_x = 4y$ $\quad$ $m_y = y$

+g means towards right.

$r_1 < r_2 < r_3$

$\frac{G4M}{r_1^2} - \frac{GM}{(d-r_1)^2}$, $\frac{G4M}{r_2^2} - \frac{GM}{(d-r_2)^2}$

$\frac{GM}{r_2^2}$ is smaller than $\frac{GM}{r_1^2}$. $\frac{GM}{(d-r_2)^2}$ is bigger than $\frac{GM}{(d-r_1)^2}$

It decreases as r increases. So, decreased with ~~B~~ r/d.

C & D options are out.

$$0 = \frac{G4M}{r^2} - \frac{GM}{(d-r)^2}$$

$\frac{GM}{(d-r)^2} = \frac{G4M}{r^2}$ $\qquad$ $\frac{GM}{(d-r)^2} = \frac{4GM}{r^2}$

~~d = 2r, r~~

~~GM~~

$r^2 > (d-r)^2$

$r > d-r$

$2r > d$

$\frac{GM}{(d-r)^2} = \frac{4GM}{r^2}$

$\frac{1}{(d-r)^2} = \frac{4}{r^2}$

$r^2 = 4(d-r)^2$

$r^2 = 4(d^2+r^2-2dr)$

$r^2 = 4d^2+4r^2-8dr$

$4d^2+3r^2-8dr=0$

$(2d \quad)(2d \quad)$

Book says answer is C, I don't know.

Date

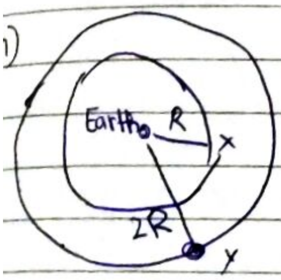
6) A planet has 3 times the mass & 3 times the radius of Earth. GFS at surface of Earth is g. What is gfs at surface of the planet.

$g = \frac{GM}{r^2}$ $\qquad a = \frac{3GM}{(3r)^2} = \frac{3GM}{9r^2} = \frac{1}{3}\frac{GM}{r^2}$

$a = \frac{1}{3}g$

A) g B) $\frac{g}{3}$ c) $\frac{g}{9}$ d) $\frac{g}{27}$

7)



What is ratio $\frac{v_x}{v_y}$ of orbital speeds of the 2 satellites?

~~$v_x = \omega R$~~ ~~$v_y = 2\omega R$~~

$\omega = \frac{2\pi}{T}$ $\qquad \omega = \frac{2\pi}{T}$

~~$\frac{v_x}{v_y} = \frac{1}{2}$~~

$\frac{mv^2}{r} = \frac{GMm}{r^2}$

$\frac{v^2}{\cancel{r}} = \frac{GM}{r^2 \cancel{r}}$ $\qquad v^2 r = GM$

$v^2 = \frac{GM}{r}$

$v^2 \propto \frac{1}{r}$

$X, \ v_x^2 \propto \frac{1}{R}$

$Y, \ v_y^2 \propto \frac{1}{2R}$

$v_x = \sqrt{\frac{GM}{R}}$ $\qquad v_y = \sqrt{\frac{GM}{2R}}$

$\frac{v_x}{v_y} = \frac{\sqrt{\frac{\cancel{GM}}{\cancel{R}}}}{\sqrt{\frac{\cancel{GM}}{2\cancel{R}}}} = \sqrt{2}$

D $\sqrt{2}$

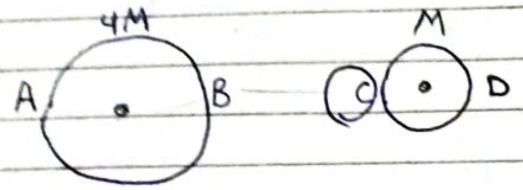
8) What is correct about probe that orbits the Earth in a circular orbit at constant speed?

A Acceleration is constant
B Velocity is constant
C Kinetic energy is constant
D Momentum is constant

Date

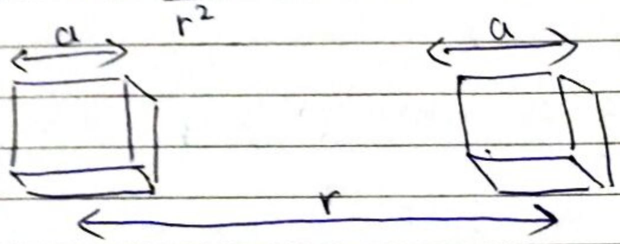
9) Two stars have masses 4M and M.

At which point is magnitude of combined grav. field least?



At C.

10) $F = \frac{Gm^2}{r^2}$ to calculate GF between two uniform cubes side a & mass m.



Result will be:

A correct

B approximately correct

C approximately correct only if $a = r$

D approximately correct only if $a \ll r$, volume can't be too much? idk?