The Dynamics of a Cantilever

Research Question: How does the vertical depression of a cantilever respond to a change in the distance at which an external force is applied to the cantilever?

Physics Internal Assessment

Introduction

Beams are an essential part of engineering. *A beam which is rigidly connected at one end to a fixed support and free to move at the other end is called a cantilever beam.*[1] Various infrastructures such as traffic lights, roofs[2], diving-boards, and bridges[3] are constructed by employing the fundamental knowledge of cantilevers.[4]

As a child, I played **Jenga**- a game which employs the concept of cantilevers to evoke children's interest. As the game progressed, more and more cantilevers[5] were made, making the game full of suspense. This curiosity rose further after **realising the importance of cantilevers in mechanical engineering** because I often watch videos of bridges, balconies and roofs collapsing just like the wooden blocks in Jenga. Hence, I decided to conduct an experiment to analyse the behaviour of a cantilever.

I wanted to test a variable which is less commonly experimented. I chose the distance at which the force is exerted on the cantilever as my independent variable. The aim of this experiment is to analyse the behaviour of a cantilever and then answer the following research question: **How does the vertical depression of a cantilever respond to a change in the distance at which an external force is applied to the cantilever?**

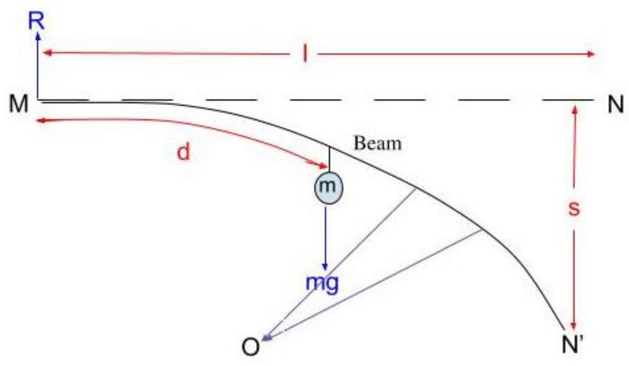

Diagram 1. Cantilever Model

Consider a cantilever, fixed at one end, M, and loaded at a distance(cm), d. The end, N, is depressed to N' and s or [N'-N] represents the vertical depression at the free end. Note that N may not be at the same height(cm) level as M. There can be an initial vertical depression before any external force is exerted on the cantilever. The force exerted downwards by load- mg (Newtons), is equally and oppositely opposed by reaction force, R, acting upwards at the supported end M. If we exert the force at a distance farther from the suspension point M than d, the vertical depression(cm) is theoretically going to increase. However, as the force is exerted for longer periods of time and as the distance increases more and more, the upper layer of the filaments at the point of suspension are more likely to get elongated while the lower layer of filaments gets compressed. This may lead to a permanent deformation of the cantilever.

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[1] Michels, W. C. (1956). *International Dictionary of Physics and Electronics*. New Jersey: Van Nostrand. Retrieved from Questia School.

[2] The Function and Aesthetics of Cantilevers. (n.d.). Retrieved August 26, 2017, from http://blog.buildllc.com/2016/02/the-function-and-aesthetics-of-cantilevers/

[3] Cantilever Bridge Facts, Design and History. (n.d.). Retrieved August 26, 2017, from http://www.historyofbridges.com/facts-about-bridges/cantilever-bridge/

[4] T. (n.d.). Cantilever. Retrieved July 28, 2017, from http://science.jrank.org/pages/1171/Cantilever.html

[5] Smith, D. (2016, November 10). How to beat anyone at Jenga. Retrieved August 26, 2017, from http://www.wired.co.uk/article/how-to-beat-anyone-at-jenga

The relationship between distance d and vertical depression s is determined below. **It is not a linear relationship, so I have used logarithms to linearize it**:

$s \propto d^n$

$\mathbf{s = kd^n}$ *(Equation 1)*

$\log(s) = \log(kd^n)$

$\log(s) = \log(d^n) + \log(k)$

$\mathbf{\log(s) = n\log(d) + \log(k)}$ *(Equation 2)*[6]

d = distance in centimetres

s = vertical depression in centimetres

n = constant

k = constant

I have decided to measure the vertical depression(cm) for the distances(cm): 10,20,30,40,50, 60,70,80 and construct a relationship between s and d, by calculating n and k for the cantilever in the experiment. I have chosen a cantilever with a low thickness (0.10cm) so that the vertical depressions in the experiment are large and observable.

This research is significant because the relationships between the distance(cm) and vertical depression(cm) of cantilevers can assist engineers to identify optimal lengths of materials for their industrial projects. Hence, it is a crucial part of mechanical engineering.

Hypothesis

There is a logarithmic linear relationship between the distance(cm) at which the force is exerted and the vertical depression(cm) on the cantilever, given that the force exerted on the cantilever remains constant. This is in the form: $\log(s) = n\log(d) + \log(k)$, or, $s=kd^n$, where s represents vertical depression (cm), d represents distance (cm), and k and n are constants.

Variables

Independent Variable: Distance of slotted-mass from cantilever's point of suspension (cm). The calibration on the beam is used to measure this variable. The slotted-mass is tied to a string of negligible mass and hung. Masking tape ensures that the string does not slide. Distance (cm) intervals are:10cm, 20cm, 30cm, 40cm, 50cm, 60cm, 70cm, 80cm.

Dependent Variable: Vertical depression of the cantilever beam (cm).

This is measured by placing a wooden ruler next to the end of the cantilever beam. Readings are taken at eye-level using a set-square once the metal beam stops vibrating.

Controlled Variables:

Table 1. Identifying and analysing controlled variables

Variable to be controlled	Why and how the variable is to be controlled
Mass of slotted-mass (g)	The force exerted on the cantilever is directly proportional to the mass of the slotted-mass because Force=(mass)*(acceleration due to gravity), where force is in Newtons and acceleration due to gravity is in ms^{-2}. Reduction in the mass will reduce the force, reducing the vertical depression and vice versa. Hence, this variable will be kept constant by using the same slotted-mass of 99.9 grams (or 0.0999 kilograms) throughout the experiment.

[6] Homer, D., & Bowen-Jones, M. (2014). *Physics: course companion*. Oxford: Oxford University Press. p17 Linearizing graphs

The metal beam (cantilever)	Any changes in the length, width, thickness and material will change the physical properties of the cantilever beam. This will affect the rate of change of vertical depression (Δs) during the experiment. Therefore, the easiest way to keep these properties controlled is by using the same beam throughout the experiment.
Length of metal beam suspended from the table	If there is an increase in the length of beam suspended, the suspended mass of the beam will increase, increasing the initial force (mg) on the beam, causing a higher vertical depression. This change in vertical depression can be mistaken as a change due to change in distance, and this reduces the accuracy and precision of the results. Therefore, a G-clamp will be firmly attached, so that the length of beam suspended remains constant throughout the experiment.

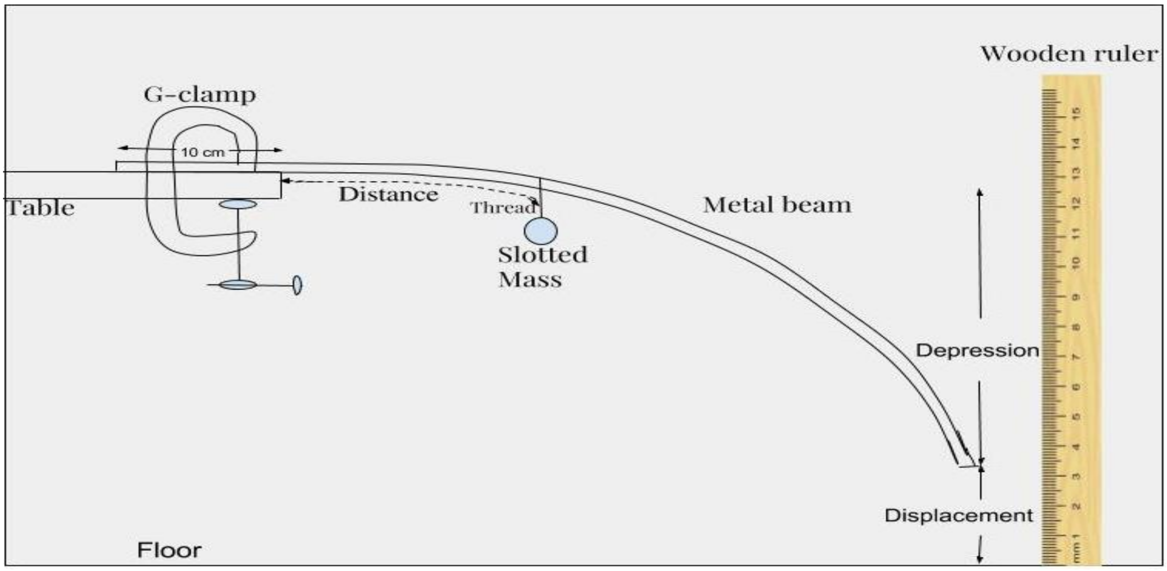
Materials

Table 2. Showing materials and properties

Material	Properties
1×Metal beam (a calibrated metal ruler)	Length: 103cm; Width: 2.80cm, Thickness: 0.10cm
Vernier Caliper (measure beam's properties)	Uncertainty: (±0.01 cm)
1×Slotted-mass	Mass: 99.9 grams
Electronic Balance	Uncertainty: (±0.1 grams)
1×Wooden Ruler	Length: 100cm; Uncertainty: (±0.05 cm)
1×G-Clamp	-
Masking Tape	-
String	-
Table or Bench	-

Method

Diagram 2. Showing the experimental set-up for the experiment



Observations and measurements before commencing

1. Ensure that the length of the metal beam on the table is 10 cm while the remaining 93 cm is suspended in air. This is easily ensured as the beam is calibrated.

2. Measure the mass of the slotted-mass using the electronic balance as it is a controlled variable.
3. Ensure that the beam is not supported by any other solids apart from the table and G-clamp.
4. Measure the initial vertical displacement using the wooden ruler. Avoid parallax error by using a set square, or any other instrument, to take eye-level reading.
5. Calculate the initial vertical depression of the beam. The initial vertical depression is the: (Displacement between the point of suspension and the floor – initial vertical displacement).

<u>Collection of Raw data</u>

6. Move the string 10 cm along the calibrated beam. Put masking tape if the string slips downwards and wait for the beam to become still.
7. Measure the vertical displacement using the wooden ruler from the end of beam to floor.
8. Measure the vertical displacement for the other distances(cm): 20,30,40,50,60,70,80.
9. Similarly collect raw data for trial 2 and trial 3 by repeating steps 6,7 and 8.

<u>Processing the Raw data</u>

10. Calculate the vertical depression using the formula:

    Vertical depression (cm) = (Displacement between the point of suspension and the floor – Vertical displacement)

Risk Assessment

There are no significant risk assessments. However, metal beam has sharp edges and should be used carefully when taking eye-level reading. The string used to hang the slotted-mass breaks frequently during the experiment, so anything placed below the set-up will be damaged. A cushion can be placed at bottom to prevent damages to the lab flooring. **This is a safe experiment with little to no environmental and ethical issues in the methodology.**

Data and Analysis

Table 3. Raw data showing distance(cm) and vertical displacement(cm)

	Vertical displacement between the end of the beam from the floor a/ cm $\Delta cm = \pm 0.1$			
Distance d/ cm $\Delta cm = \pm 0.05$	Trial 1	Trial 2	Trial 3	Mean Displacement a/ cm
0.0	*44.5*	*44.5*	*44.5*	44.5 ± 0.0
10.0	43.7	44.3	42.6	43.5 ± 0.9
20.0	41.5	41.8	40.2	41.2 ± 0.8
30.0	39.0	38.6	38.0	38.5 ± 0.5
40.0	36.0	35.6	34.9	35.5 ± 0.6
50.0	33.2	32.2	31.8	32.4 ± 0.7
60.0	28.1	27.7	27.1	27.6 ± 0.5
70.0	26.1	25.9	25.1	25.7 ± 0.5
80.0	24.5	24.0	23.5	24.0 ± 0.5

All data in Table 3 except the uncertainty for the distance column has been formatted to 1 decimal place. The uncertainty of ±0.05 is important for calculating the uncertainty of the trials in Table 4 and hence has not been formatted to 1 decimal place.

<u>Data Calculation</u>

Example 1

The mean vertical displacement for row 2 in Table 3 was calculated using the formula:

$$\frac{(a_1 + a_2 + a_3)}{3}$$

$= \frac{(43.7 + 44.3 + 42.6)}{3}$

$= 43.5\underline{3}$ cm

= 43.5 cm (1 decimal place)

Calculation of uncertainty of mean vertical displacement for row 2:

$$\frac{(Range)}{2}$$

$= \frac{(44.3 - 42.6)}{2}$

$= \pm0.8\underline{5}$ cm

$= \pm0.9$ cm (1 decimal place)

Table 4. Processed data showing distance(cm) and vertical depression(cm)

	Vertical depression of the beam s/ cm $\Delta cm = \pm0.1$			
Distance d/ cm $\Delta cm = \pm0.1$	Trial 1	Trial 2	Trial 3	Mean Depression s/ cm
0.0	*46.3*	*46.3*	*46.3*	*46.3* $\pm$ *0.0*
10.0	47.1	46.5	48.2	47.3 $\pm$ 0.9
20.0	49.3	49.0	50.6	49.6 $\pm$ 0.8
30.0	51.8	52.2	52.8	52.3 $\pm$ 0.5
40.0	54.8	55.2	55.9	55.3 $\pm$ 0.6
50.0	57.6	58.6	59.0	58.4 $\pm$ 0.7
60.0	62.7	63.1	63.7	63.2 $\pm$ 0.5
70.0	64.7	64.9	65.7	65.1 $\pm$ 0.5
80.0	66.3	66.8	67.3	66.8 $\pm$ 0.5

<u>Data Calculation</u>

Example 2

The vertical depression for the first trial and first row was calculated using the formula:

(Displacement between the point of suspension and the floor) – (Vertical displacement)

= 90.8 – 44.5

= 46.3 cm

Calculation of uncertainty for the vertical depression of the beam:

$= (\pm0.05) + (\pm0.05)$

$= \pm0.1\underline{0}$

$= \pm0.1$ cm (1 decimal place)

Example 3

The mean vertical depression for second row in Table 4 was calculated using the formula:

$= \frac{(s_1 + s_2 + s_3)}{3}$

$= \frac{(47.1 + 46.5 + 48.2)}{3}$

$= 47.2\underline{7}$ cm

= 47.3 cm (1 decimal place)

Calculation of uncertainty of mean vertical depression for row 2:

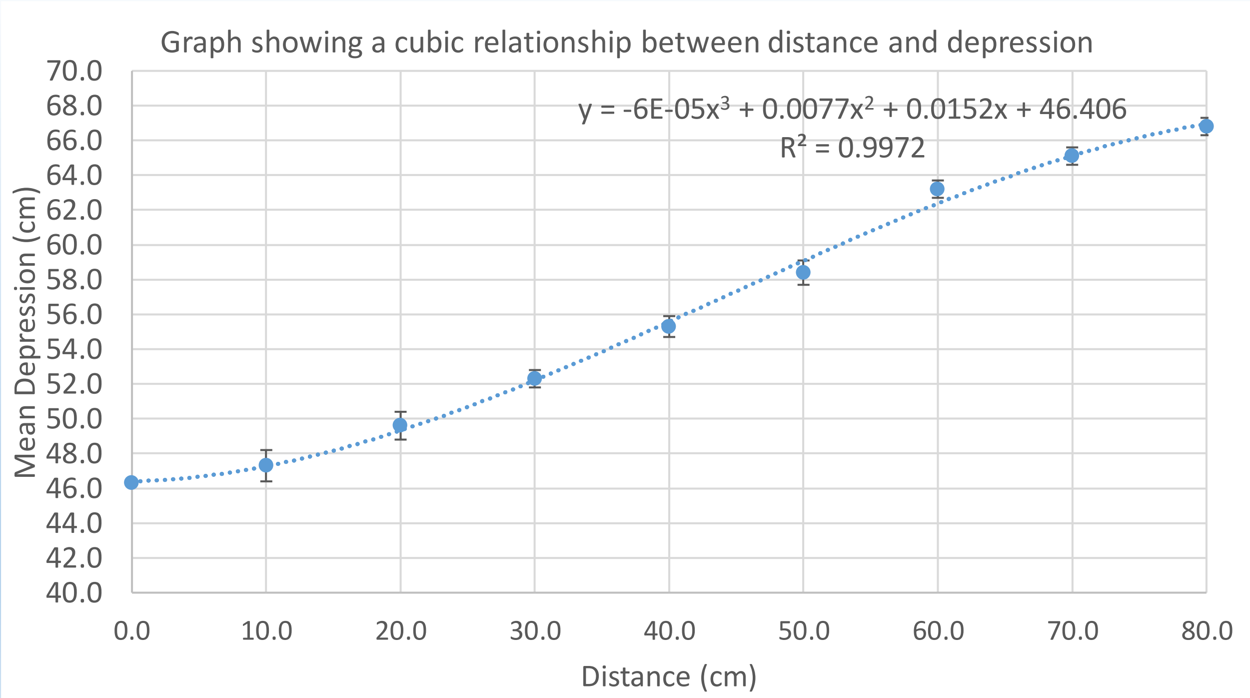
$\frac{\text{Range}}{2} = \frac{(48.2 - 46.5)}{2}$

$= \pm 0.8\underline{5}$ cm

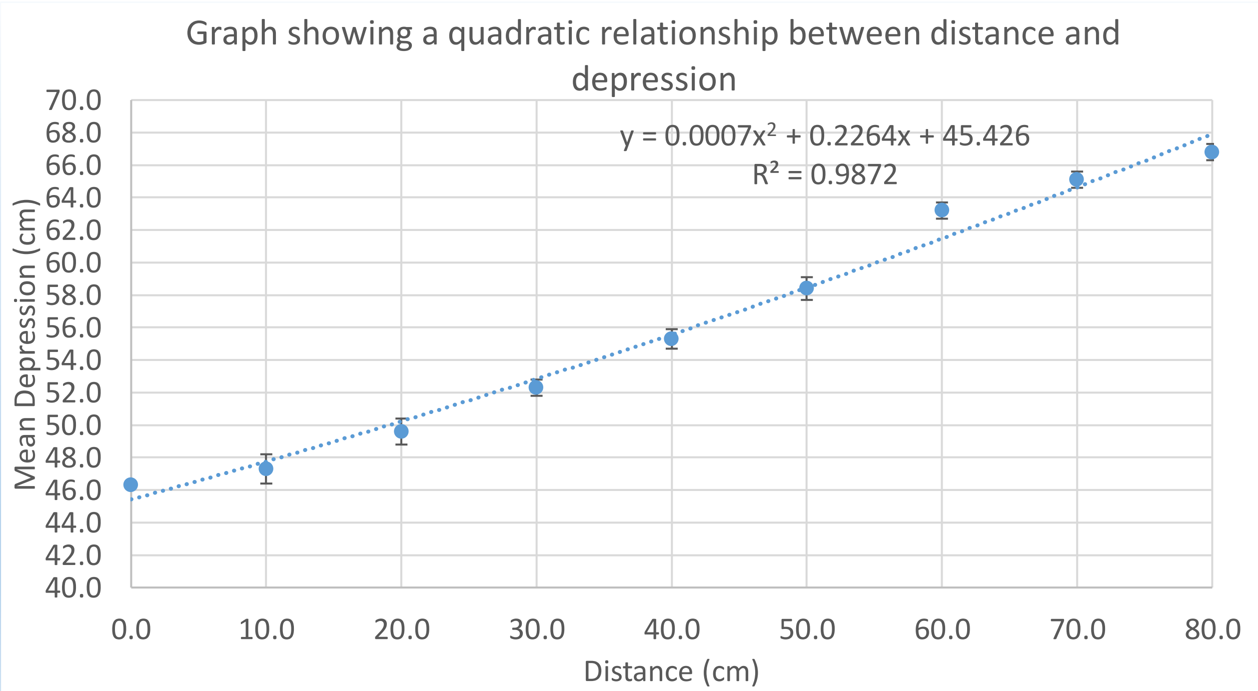
$= \pm 0.9$ cm (1 decimal place)

Graphical Analysis

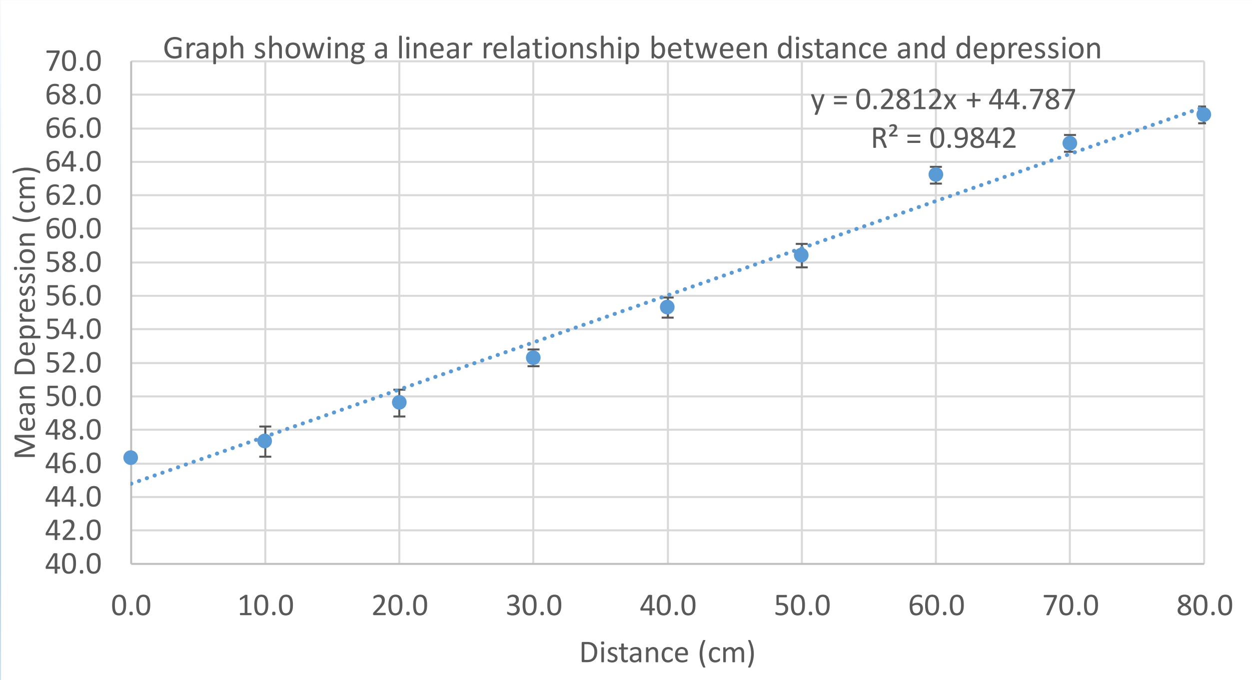
Graph 1. Showing a cubic relationship between distance and mean depression



Graph 2, showing a quadratic relationship between distance and mean depression



Graph 3, showing a linear relationship between distance and mean depression



Data collected and graphs support a cubic relationship because Graph 1 intersects 8 out of 9 plotted error bars (Distance (cm) = 0, 10,20,30,40,50,70,80), whereas Graph 2 intersects 6 out of 9 plotted error bars (Distance (cm) = 10,20,30,40,50,70) and Graph 3 intersects 4 (Distance (cm) = 10,20,50,80). This is contradictory[7] to the hypothesis. The reason for the cubic relationship could be the deformation of the metal beam during the experiment. However, this needs to be proven.

Data in Table 4 suggests that deformation has taken place. This is because range for the final 6 distances is derived using the formula: (Trial 3 - Trial 1), as the 3rd trial consistently had the largest magnitude and the 1st trial had the lowest. There is an increasing trend in the magnitude of vertical depressions as the trials are carried out.

Table 5. showing evidence of deformation on the beam

Distance d/ cm $\Delta cm = \pm 0.1$	Vertical depressions of the beam for the three trials s/ cm $\Delta cm = \pm 0.1$ Trial 1 < Trial 2 < Trial 3
30.0	51.8 < 52.2 < 52.8
40.0	54.8 < 55.2 < 55.9
50.0	57.6 < 58.6 < 59.0
60.0	62.7 < 63.1 < 63.7
70.0	64.7 < 64.9 < 65.7
80.0	66.3 < 66.8 < 67.3

[7] Gleason, R. E. (n.d.). *On the Complete Logarithmic Solution of the Cubic Equation*[PDF]. Annals of Mathematics.

Also, when the distance (cm) is 10.0 and 20.0, the largest vertical depression is in Trial 3. Therefore, Trial 3 has the largest vertical depression in all 8 trials and Trial 1 has the lowest vertical depression in 6 consecutive data recordings. It can now be concluded that the beam was an inelastic body because it did not return to its original state after deformation. This led to results which support a cubic relationship.

Since deformation is proven, a logarithmic linear relationship can be constructed, in line with the hypothesis, between distance and mean vertical depression:

log(s) = nlog(d) + log(k)

d = distance

s = mean vertical depression

n = constant (gradient)

k = constant (log(k) is y-intercept)

Table 6. converting the data into logarithmic form

Distance d/ cm $\Delta cm = \pm 0.10$	Log (d)	Mean Depression s/ cm	Log (s)
10.000	1.000	47.300 ± 0.900	1.675 ± 0.008
20.000	1.301	49.600 ± 0.800	1.695 ± 0.007
30.000	1.477	52.300 ± 0.500	1.718 ± 0.004
40.000	1.602	55.300 ± 0.600	1.743 ± 0.005
50.000	1.699	58.400 ± 0.700	1.766 ± 0.005
60.000	1.778	63.200 ± 0.500	1.801 ± 0.003
70.000	1.845	65.100 ± 0.500	1.814 ± 0.003
80.000	1.903	66.800 ± 0.500	1.825 ± 0.003

Data has been formatted to 3 decimal places because the maximum place value for the uncertainties is *thousandths*[8].

<u>Data calculation</u>

Calculating uncertainty in logarithmic calculations:

Log(s) = $\log_{10}$(s)

$\pm$log(s) = uncertainty of log(s)

$\pm$s = uncertainty of s

$\pm$**log(s) = *0.434*** $\times$ $\left(\frac{\pm s}{s}\right)$ (*Equation 3)*[9]

Example 4

Calculation of log (s) and its uncertainty for row 1 in Table 6:

Log (s) = $\log_{10} 47.300 \pm 0.434 \times \left(\frac{0.900}{47.300}\right)$

Log (s) = $1.674\underline{9} \pm 0.008\underline{26}$ (3 decimal places)

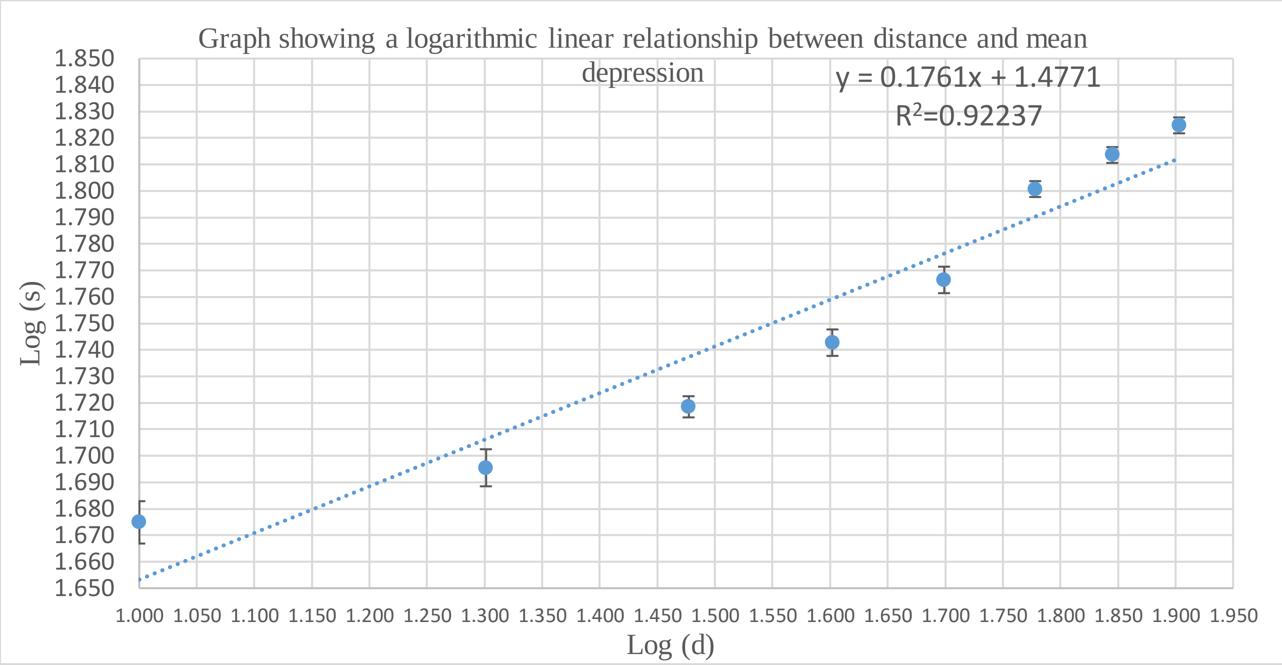
Log (s) = 1.675 ± 0.008

The equation that defines the relationship between distance and vertical depression of the cantilever used in this experiment can be derived by plotting a graph.

[8] Place Value. (n.d.). Retrieved August 05, 2017, from http://www.enchantedlearning.com/math/decimals/placevalue/

[9] *Error Propagation in Arithmetic Calculations*[PDF]. (n.d.). https://terpconnect.umd.edu/~toh/models/ErrorPropagation.pdf

Graph 4, showing a logarithmic linear relationship between distance and mean depression



$\log(s) = n\log(d) + \log(k)$

$\log(k) = 1.477$ (y-intercept)

$n = 0.176$ (gradient)

$\mathbf{\log(s) = 0.176\log(d) + 1.477}$ *(Equation 4)*

$s = kd^n$

$k = 10^{1.477} = 30.0$

$\mathbf{s = 30.0d^{0.176}}$ *(Equation 5)*

The line of best-fit in Graph 4 does not intersect any plotted error bars. At first, this line was manually steepened to intersect more error bars. However, since the previous lines of best-fit were auto-generated by the software, this line was left unaltered. **Furthermore, worst fit lines do not intersect most points either, so calculating uncertainty in the equation would not be useful**. However, the **equation is verified below** by calculating the percentage difference between measured and calculated vertical depressions. Be the differences negligible, the equation is accepted, and vice-versa.

Table 7. Comparing the measured and calculated data

Distance d/ cm	Measured depression s_1/cm	Calculated depression s_2/cm $s_2 = 30.0d^{0.176}$	Difference z/cm $z = \lvert s_1 - s_2 \rvert$	% Difference z'/% $z' = (\frac{z}{s_1})(100)$
10	47.3 ± 0.9	45.0	2.30	4.86%
20	49.6 ± 0.8	50.8	1.20	2.42%
30	52.3 ± 0.5	54.6	2.30	4.40%
40	55.3 ± 0.6	57.4	2.10	3.80%
50	58.4 ± 0.7	59.7	1.30	2.23%
60	63.2 ± 0.5	61.7	1.50	2.37%
70	65.1 ± 0.5	63.4	1.70	2.61%
80	66.8 ± 0.5	64.9	1.90	2.84%

The percentage differences are negligible (largest is 4.86%). z is very small compared to s_1. Hence, the data collected supports the equation and the equation is accepted.

Conclusion

This was an experiment to apply the cantilever theory on an actual cantilever and determine the relationship between distance(cm) and vertical depression(cm).

After collecting and processing the data from the experiment, it can be concluded that the **results support the hypothesis**. "Given that the force exerted on the cantilever remains constant", there is evidence of a "logarithmic linear relationship between the distance(cm) at which the force is exerted and the vertical depression(cm) on the cantilever". First, the mean vertical displacement was measured, stated in Table 3, which was then used to calculate mean vertical depression in Table 4. Then, the relationship between distance(cm) and vertical depression(cm) was determined using Graphs 1,2 and 3. The data was converted into logarithmic form in Table 6 and plotted in Graph 4 where variables n and k, also proposed in the hypothesis, were calculated using the trend-line in Graph 4. The following equations were formed:

$$\mathbf{\log(s) = 0.176\log(d) + 1.477} \quad \textbf{\textit{(Equation 4)}}$$

$$\mathbf{s=30d^{0.176}} \quad \textbf{\textit{(Equation 5)}}$$

The equations are rearrangements of each other. Random and systematic errors led to the line of best fit in Graph 4 not intersecting any error bars. This is largely due to deformation of the cantilever that is also listed in introduction- "This may lead to a permanent deformation of the cantilever.", and is proven using Table 5. **A modification in the methodology, which is stated in the Evaluation, can help reduce this error.** Nevertheless, this equation was verified in Table 7 by comparing the magnitudes of vertical depression calculated using the equation, with the magnitudes of mean vertical depression measured in the experiment. The negligible percentage differences (highest being 4.86% and lowest being 2.23%) between the calculated and measured vertical depressions support the acceptance of this equation.

Hence, **the research question: "How does the vertical depression of a cantilever respond to a change in the distance at which an external force is applied to the cantilever?" was worthy of investigation and is answered**: As theory answers it qualitatively, a cantilever was tested to answer this question quantitatively. The response of vertical depression of the cantilever to a change in the distance at which an external force is applied to the cantilever is in the form: $s=30d^{0.176}$, or, $\log(s) = 0.176\log(d) + 1.477$, where s represents vertical depression in centimetres, d represents distance in centimetres (10,20,30,40,50,60,70,80).

Evaluation

Further research suggestions

The relationship between distance and vertical depression can be calculated for diving boards to help divers determine where they should jump from on the diving board based on their mass. This captures my interest because I will be able to apply Physics to real-life situations.

Weaknesses, improvement and strengths

The experiment was acceptably accurate and precise. While it was conducted within the parameters of a school lab, some improvements in the methodology and investigation can improve the accuracy of this research.

Table 8. Possible systematic errors

Source of error and its effect	**Significance & evidence**	**Improvements**
<u>Systematic errors affecting accuracy</u>		
Beam deformation: The deformation of the metal beam increased the magnitude of vertical depression as the experiment progressed.	High significance because the beam was supposed to be a controlled variable. As it got deformed, the vertical depression increased as the time of the experiment increased. The precision and accuracy of the data and the equations decreased. Evidence is shown in Table 5.	Deformation is a natural phenomenon. But, a change in methodology so that initial vertical depression (when distance(cm) = 0) is calculated before each trial, instead of only at the start of the experiment, can reduce the error. For example- if the initial vertical depression for Trial 2 is significantly higher than Trial 1, it can be inferred that the beam is deformed and it can be replaced by a new identical beam before continuing the experiment. Thicker beams can be used as they are unlikely to deform as much the beam used.
<u>Random errors affecting precision</u>		
String length: The string used to hang slotted-mass from the beam broke thrice and the length of the string was not kept constant. Furthermore, the masking tape obscures vision from seeing that the string is placed at the right point.	Low significance because the force exerted is [mass × gravity] and the length of string is not a factor. The masking tape is necessary to prevent the string from sliding.	Use a string that is not made of a series of intertwined and twisted fibres because these fibres can break and weaken the string. Groove the beam at points where the mass is to be hung. This prevents string from sliding. Or, scotch tape can be used for transparent vision.
Wooden ruler calibration: It is calibrated to 1 decimal place which creates an uncertainty of 0.05cm in every reading. So, the actual vertical displacement could be higher or lower.	Low significance because an uncertainty of 0.05cm is small. (less than 1% percentage uncertainty). So, precision is barely affected.	An increase in the number of trials can reduce the significance of this error.
Parallax error: Although readings were taken at eye-level, there are differences between the apparent and real magnitude of readings. So, the actual reading could be higher or lower.	High significance as we cannot quantify the uncertainty this factor has caused. It contributes to inaccuracy in measurements.	Take eye-level readings from a fixed point at a fixed distance to prevent changes in the apparent magnitudes of readings. This way, the readings will have higher precision. To improve accuracy, use inch tape instead of a wooden ruler at it is more adjustable.

Table 10. Possible strengths and their effects

Strength	**Effect**
Beam with the smallest magnitude of thickness, 0.10cm, was chosen.	The beam's Δvertical depression and Δvertical displacement was higher than other beams in the lab. This reduced the proportion between errors and vertical depressions, making the results more precise.
String was used to hang the slotted-mass instead of sticking the mass to the beam.	If the slotted-mass was stuck to the beam, the force would spread across the area of slotted-mass that is in contact with the beam instead of being applied only at the distance required. The string has a negligible surface area , making it easier to make sure that the force is only applied at the distance required.

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