SALLYWEATHERLY

CHEAT SHEET
FOR MATHS SKILLS IN IB PHYSICS

All IB Physics students should be able to meet the following mathematical requirements. Tick the box when you feel you understand each point.

1 PERFORM THE BASIC ARITHMETIC FUNCTIONS: ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION

It's good to start with the easy stuff! ;-)

ADDITION: $12 + 6 = 18$ ☐

SUBTRACTION: $81 - 3 = 78$ ☐

MULTIPLICATION: $20x3 = 60$ ☐

Note: Taking 75.34 as an example, In some countries, the point located between two numbers indicates the product of 75 and 34 (e.g. $75x34$) - this notation is not used in IB exams. In IB exams, 75.34 indicates a separation between the whole number and the fraction.

DIVISION: $\frac{100}{4} = 25$ ☐

Note: You may see the following symbols to indicate division throughout this course: 26÷2, 26/2 or 26:2.

2 CARRY OUT CALCULATIONS INVOLVING: MEANS, DECIMALS, FRACTIONS, PERCENTAGES, RATIOS, APPROXIMATIONS AND RECIPROCALS

MEAN: $\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$ ☐

WHAT? Please make this more simple…

Example:

What is the mean of these numbers?

$$6, 11, 7$$

- Add the numbers: $x_1 + x_2 + x_3 = 6 + 7 + 11 = 24$
- Divide by how many numbers (there are 3 numbers): $\bar{x} = \frac{24}{3} = 8$

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DECIMALS: In Britain and the US, the decimal point is used to define a decimal fraction. The point is used to separate the whole number from the fraction. In some other countries, the comma is used to separate the whole number from the fraction.

Example:

Britain and US: 4.56
Other notation: $4{,}56$

IB exams use the decimal point, not a comma.

FRACTIONS:

It is easy to add fractions with the same denominator: $\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

If the denominators are different, we somehow make them the same:
$\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{2}{8} = \frac{5}{8}$
(In this example $\frac{1}{4}$ has an equivalent fraction of $\frac{2}{8}$, which is the same denominator)

PERCENTAGES:

Here's a little trick to make percentage calculations (without a calculator in Paper 1!) easier:

x% of y = y% of x

Example:

8% of 50
8% of 50 is the same as 50% of 8
And 50% of 8 is 4
So 8% of 50 is 4

This blew my mind when I realised it!

RATIOS

Top Tips on Ratios:

- A ratio is in its simplest form when both sides are whole numbers and there is no whole number by which both sides can be divided.
- When scaling ratios up or down, always remember that the same unit of measurement must be applied to both sides. Also bear in mind that the rules of the original ratio must be upheld.
- When calculating the ratio between different orders of magnitude, simply subtract the powers of ten.

APPROXIMATIONS:

Sometimes you are gonna have to approximate. You get used to it and there is usually one multiple choice question in Paper 1 that will require you to have an idea of power of magnitude.

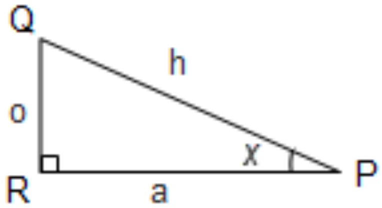
I recommend you have a good look through this website: http://scaleofuniverse.com

3 CARRY OUT MANIPULATIONS WITH TRIGONOMETRIC FUNCTIONS

This sounds hards, but it’s not.

All it means is that you have to be able to use the sine, cosine and tangent functions on your calculator. Most calculators have buttons to find the sin, cos and tan of an angle. You would usually have to set the calculator to degrees mode.

You also need to be happy with the SOH CAH TOA memory aid to remember the ratios for the sine, cosine and tangent functions.



Consider the right triangle above.

- a is the length of the side adjacent to the angle (x) in question.
- o is the length of the side opposite the angle.
- h is the length of the hypotenuse.
- "x" represents the measure of the angle in degrees (or radians).

$$sin(x) = \frac{o}{h}$$

$$cos(x) = \frac{a}{h}$$

$$tan(x) = \frac{o}{a}$$

For each of the six functions there is an inverse function that works in reverse. On calculators, the inverse functions are sometimes written $cos^{-1}(x)$.

4 CARRY OUT MANIPULATIONS WITH LOGARITHMIC AND EXPONENTIAL FUNCTIONS (**HL ONLY**)

Logarithms are the "opposite" of exponentials, just as subtraction is the opposite of addition.

Logs "undo" exponentials. Technically speaking, logs are the inverse of exponentials.

Think of logs like this:

$y = b^x$ is the same as $log_b(y) = x$

Example:

Evaluate $log_{10} = 1000$

So we ask ourselves 3 to what power is 1000?

$10^3 = 1000$
$log_{10}1000 = 3$

In words, we would say the logarithm is 3. What we have really been talking about here is an inverse. The inverse of the logarithm of base 10 is 10 to the logarithm.

Hope this makes sense???

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5 USE STANDARD NOTATION

Physics goes from the microscopic values of atomic and nuclear physics to the macroscopic values of astrophysics. We do not want to be writing that the mass of an electron is 0.000000000000000000000000000091kg….

Instead, it is easier to write $9.1x10^{-31}kg$

A number if standard form written in two parts:

1. Just the digits (with the decimal point placed after the first digit), followed by
2. $x10$ to a power that puts the decimal point where it should be (i.e. it shows how many places to move the decimal point).

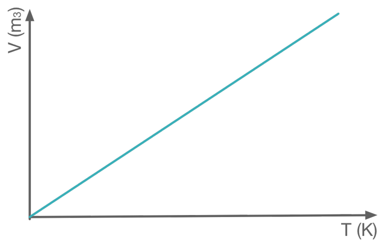
$1546.7 = (Part\ 1)1.5467\ x\ (Part\ 2)10^3$

The EXP button on your calculator stands for $x10$. If you want to type in $9x10^3$, then press the buttons 9 EXP 3

6 USE DIRECT AND INVERSE PROPORTION

DIRECT PROPORTION:

Two variables are in direct proportion if you draw a graph and it has a straight line through the origin, like this:



Mathematically, you would express the relationship as:

$V \alpha T$

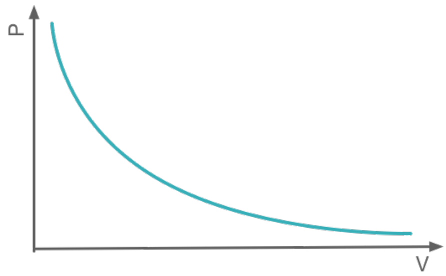
It can also be expressed as:

$V = kT$

Which is likened to the equation of a straight line ($y = mx + c$), where there is no y-intercept.

INDIRECT PROPORTION:

Two variables are indirectly proportional if you draw a graph and looks like this:



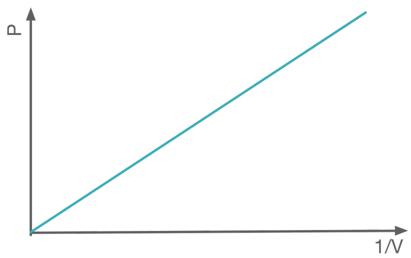
Mathematically, you would express the relationship as:

$$P \alpha \frac{1}{V}$$

It can also be expressed as:

$$P = k\frac{1}{V}$$

Which is likened to the equation of a straight line ($y = mx + c$), where there is no y-intercept and the x-axis has the variable $\frac{1}{V}$.



7 SOLVE SIMPLE ALGEBRAIC EQUATIONS

The best way to solve an equation is by using 'inverses', or undoing what the equation is doing.

To use this method to solve equations remember that:

- Adding and subtracting are the inverse (or opposite) of each other.
- Multiplying and dividing are the inverse of each other.

Example:

Solve $2a + 3 = 7$

1. Undo the $+3$ by subtracting 3. Remember, you need to do it to BOTH sides, $2a + 3(-3) = 7(-3)$
2. Undo the multiply by 2 by dividing by 2, again on both sides, $\frac{2a}{2} = \frac{4}{2}$

$$\underline{a = 2}$$

8 SOLVE LINEAR SIMULTANEOUS EQUATIONS

Follow these simple rules:
1. Multiply each equation by a suitable number so that the two equations have the same leading coefficient.
2. Subtract the second equation from the first.
3. Solve this new equation for y and find the value of y.
4. Substitute the value of y into either original equation

Example:

Equation 1: $2x + 3y = 8$
Equation 2: $3x + 2y = 7$

1. Multiply each equation by a suitable number so that the two equations have the same leading coefficient.

$$3 * (2x + 3y = 8) = 6x + 9y = 24$$
$$2 * (3x + 2y = 7) = 6x + 4y = 14$$

2. Subtract the second equation from the first

$$5y = 10$$

3. Solve the equation for y and find the value of y.

$$y = \frac{10}{5} = 2$$

$$\underline{y = 2}$$

4. substitute the value of y into either original equation

$$2x + 3(2) = 8$$
$$\underline{x = 1}$$

9 PLOT GRAPHS (WITH SUITABLE SCALES AND AXES) INCLUDING TWO VARIABLES THAT SHOW LINEAR AND NON-LINEAR RELATIONSHIPS

Rules of graph drawing:

- Scales should be chosen so that the plotted points occupy at least half of the graph grid in both x and y directions
- You must label each axis with the quantity the is being plotted
- The scale direction must be conventional (i.e. increasing from left to right)
- Choose scales that are easy to work with (e.g. go up in 5's or 10's. NOT 3's or 7's)
- Label the scales frequently. You should not have more than three large scales without a numerical label.
- All plotted points must be inside the graph (not outside the margin area)
- All observations must be plotted

10 INTERPRET GRAPHS, INCLUDING THE SIGNIFICANCE OF GRADIENTS, CHANGES IN GRADIENTS, INTERCEPTS AND AREAS.

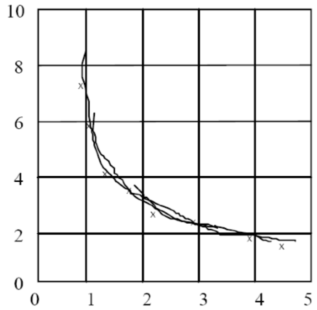
Here's a quick round up of some if the graphs you might meet in IB physics and what the gradient represents:

Type of Graph		**Gradient Represents :**	**Area Represents:**
Y-Axis	**X-Axis**		
Distance	Time	Speed	
Displacement	Time	Velocity	
Velocity	Time	Acceleration	Displacement
Speed	Time	Acceleration	Distance
Force	Acceleration	Mass	
Force	Distance	Work Done	
Force	Extension	Work Done (in extending spring)	
Force	Time	Impulse / Change of Momentum	
$sin\theta_1$	$sin\theta_2$	Refractive Index	
Potential Difference	Current	Internal Resistance	

11 DRAW LINES (EITHER CURVES OR LINEAR) OF BEST FIT ON A SCATTER PLOT GRAPH

Quick round up on this:

- There must be a reasonable balance of points around the line of best fit.
- Use a clear plastic ruler (where possible) to find the line of best fit.
- The line must be thin and clear. No hairy of thick lines are allowed.



Not acceptable - 'hairy' curve

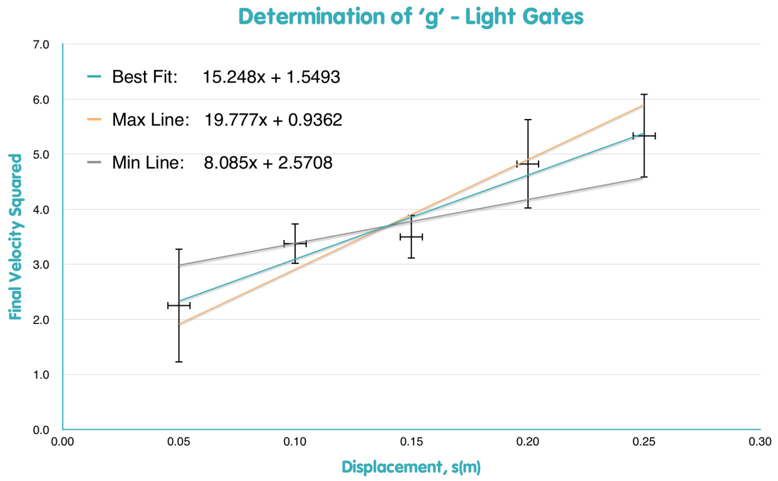
12 ON A BEST-FIT LINEAR GRAPH, CONSTRUCT LINEAR LINES OF MAXIMUM AND MINIMUM GRADIENTS WITH RELATIVE ACCURACY (BY EYE) TAKING INTO ACCOUNT ALL UNCERTAINTY BARS.

Follow these steps :

1. Draw the "best" line through all the points, taking into account the error bars. Measure the slope of this line.
2. Draw the "min" line -- the one with as small a slope as you think reasonable (taking into account error bars), while still doing a fair job of representing all the data. Measure the slope of this line.
3. Draw the "max" line -- the one with as large a slope as you think reasonable (taking into account error bars), while still doing a fair job of representing all the data. Measure the slope of this line.
4. Calculate the uncertainty in the slope as one-half of the difference between max and min slopes.

Example:

Consider this graph below (note that all lines pass through "area" subtended by the uncertainty bars):



Following the steps outlined above:

1. The gradient of the best fit line is: $m_{best} = 15.248 m\,s^{-2}$
2. The gradient of the minimum best line is: $m_{min} = 8.085 m\,s^{-2}$
3. The gradient of the maximum best line is: $m_{max} = 19.777 m\,s^{-2}$
4. Uncertainty = $\dfrac{m_{max} - m_{min}}{2} = 5.846 m\,s^{-2}$

Final Gradient: $15.248 \pm 5.846 m\,s^{-2}$

13 INTERPRET DATA PRESENTED IN VARIOUS FORMS (FOR EXAMPLE, BAR CHARTS, HISTOGRAMS AND PIE CHARTS)

BAR CHARTS:
A bar chart is made up of columns plotted on a graph. Here is how to read a bar chart.

- The columns are positioned over a label that represents a categorical variable.
- The height of the column indicates the size of the group defined by the column label

HISTOGRAMS:
Like a bar chart, a histogram is made up of columns plotted on a graph. Usually, there is no space between adjacent columns. Here is how to read a histogram.

- The columns are positioned over a label that represents a quantitative variable.
- The column label can be a single value or a range of values.
- The height of the column indicates the size of the group defined by the column label.

14 REPRESENT ARITHMETIC MEAN USING X-BAR NOTATION (FOR EXAMPLE, $\bar{x}$)

MEAN: $\bar{x} = \dfrac{x_1 + x_2 + \ldots + x_n}{n}$

15 EXPRESS UNCERTAINTIES TO ONE OR TWO SIGNIFICANT FIGURES, WITH JUSTIFICATION.

When recording raw data, estimated uncertainties should be indicated for all measurements.

There are different conventions for recording uncertainties in raw data.

- The simplest convention is the least count, which simply reflects the smallest division of the scale, for example ±0.01 g on a top pan balance.
- The instrument limit of error is usually no greater than the least count and is often a fraction of the least count value. For example, an analogue ammeter is often read to half of the least count division, which would mean that a value of 23 mA becomes 23.0 mA (±0.5 mA). Note that the value is now cited to one extra decimal place so as to be consistent with the uncertainty.
- The estimated uncertainty takes into account the concepts of least count and instrument limit of error but also, where relevant, higher levels of uncertainty as indicated by an instrument manufacturer.

If uncertainties are small enough to be ignored, the student should note this fact. In addition, students can make educated guesses about uncertainties depending on the method of measurement.

All uncertainties should be given to one or two significant figures.

That's it folks!

I hope you have found this cheat sheet useful? If so, have a look on my website www.sallyweatherly.com for more great giveaways for IB Physics Students.

Find more great resources for IB Physics on:
sallyweatherly.com