Option C Imaging

Date

C1 Introduction to imaging

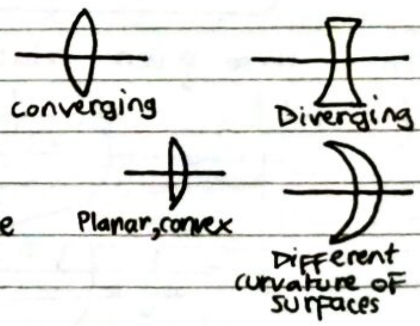
- Lenses and mirrors
- Lens defect : Spherical and chromatic aberration

Some formulas are derived in question-answers.

L.O.
- Converging & diverging lenses
- Concave & convex mirrors
- Ray diagrams
- Difference between real & virtual images
- Linear & angular magnifications
- Spherical & chromatic aberrations

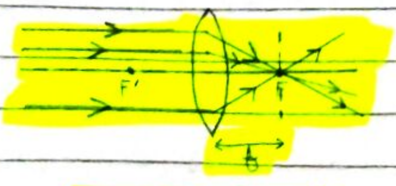
C1.1 Lenses

- The passage of light rays through lenses depends on refractive index of lens, radii of two spherical surfaces of lens and angle of incidence of ray. There is deviation.
- We assume the lens is always very thin.
- Principal axis – the straight line through the centre of the lens at right angle to the lens' surface.

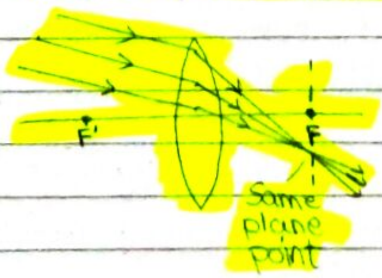


C1.2 Converging Lenses

- The ray of light changes its direction towards the axis of the lens.
- Rays parallel to principal axis will refract and pass through focal point.
- If not parallel, they still intersect at a point in the same plane as focal point.
- Distance between focal point and centre of lens is focal length, f.



Standard Ray 1



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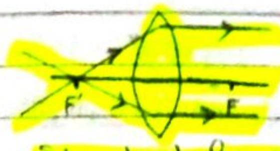
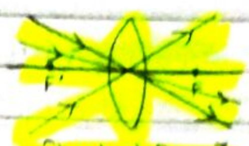


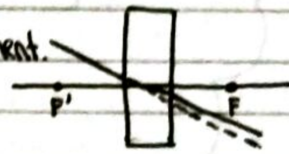
Standard Ray 2



Standard Ray 3

This ray goes undeflected when directed towards centre of lens.

- Undeflected ray : At midpoint of lens, the two lens surfaces are almost parallel. The ray should just get shifted parallel to itself. But, since lens is thin, we ignore the ~~shifting~~ displacement.



Exam tip

| Power = $\frac{1}{\text{focal length}}$ |

Dioptres, D = $1 \, m^{-1}$

If $f = 25cm$

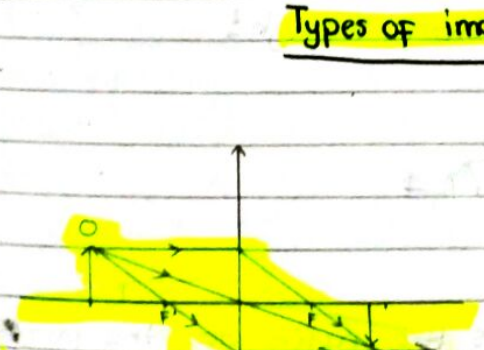
Power = $\frac{1}{0.25} = 4$ Dioptres

Moving on...

So, we get different types of images depending on distance of object from the lens.

 = Easier way to draw any lens

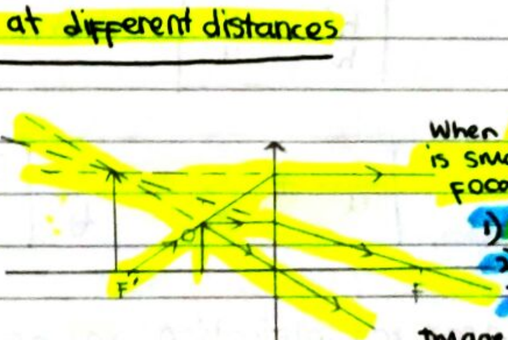
Types of images formed at different distances



When distance of object is more then focal length

1) Inverted 2) Smaller 3) Real

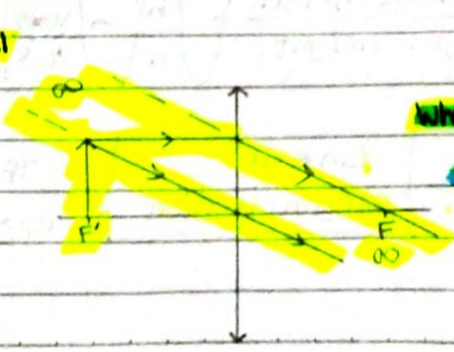
~~Object~~ Image distance and height ↓



When object dis. is smaller than focal length.

1) Upright
2) Magnified
3) Virtual

Image distance and height ↑



When object distance = focal length, Refracted rays are parallel. The rays meet at infinity.

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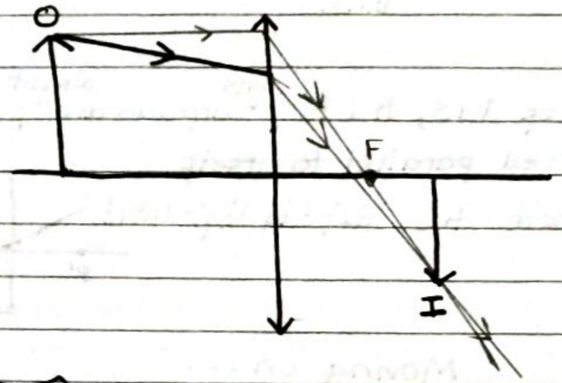
Real image = Formed by actual rays and can be projected on a screen.
Virtual image = Formed by extensions of rays and cannot be projected on a screen.

Example



a) Draw a line to show how the ray refracts.

b) Draw another line to locate focal length.

This is the graphical method for finding image.

There is a algebrical method also.

h = height of object
h' = height of image
u = distance from object
v = distance from image
f = focal length

$$\frac{h'}{h} = \frac{v}{u}$$ Not the main equation though!

- $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$ Thin lens equation

- Linear magnification, m, of the lens is the ratio of the image height to the object height:

~~$m = \frac{\text{image height}}{\text{object height}} = \frac{h'}{h} =$ Linear magnification~~

$$m = \frac{h'}{h} = -\frac{v}{u}$$ Linear Magnification

The minus sign is just more appropriate.

small m.

M is angular magnification.

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Why - sign is important?

The thin-lens equation and magnification formula have conventions that must be ~~plo~~ followed:

1) f is positive in converging lens.

2) u is positive.

Remember them!

3) v is positive for real images. (formed on other side of object).

4) v is negative for virtual images. (same side as object).

5) $M > 0$ means image is upright.

6) $M < 0$ means image is inverted.

7) $|M| > 1$ means image is larger than object.

8) $|m| < 1$ means image is smaller than object.

Examples

Q) A converging lens has focal length 15 cm. An object is placed 60 cm from lens. Determine size of image and magnification of lens.

$$-\frac{v}{u} = \frac{h'}{h}$$

The negative sign means image is inverted. Image is 3 times shorter than the object.

$$\frac{1}{15} = \frac{1}{60} + \frac{1}{v}$$

$$\frac{1}{20} = \frac{1}{v}, \quad v = 20 \text{ cm}$$

$$-\frac{20}{60} = \frac{h'}{h} = M = -\frac{1}{3}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{60} + \frac{1}{v} = \frac{1}{15}$$

$$\frac{1}{v} = \frac{1}{20}$$

$$v = 20 \text{ cm}$$

$$-\frac{20}{60} = -\frac{1}{3}$$

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Example

An object is placed 15 cm in front of converging lens of focal length 20cm. Determine size of image and value of magnification.

Sol. Image will be virtual. So v is negative. (Intuition)

$$\frac{1}{f} = \frac{1}{u} + \left(\frac{1}{-v}\right)$$

$$\frac{1}{20} = \frac{1}{15} + \left(\frac{1}{-v}\right)$$

$$-\frac{1}{v} = \frac{4-3}{60}$$

$$-\frac{1}{v} = \frac{1}{60}$$

$$v = -60 \text{ cm}$$

It is indeed negative.

$$m = -\left(\frac{-60}{15}\right)$$

$$m = 4$$

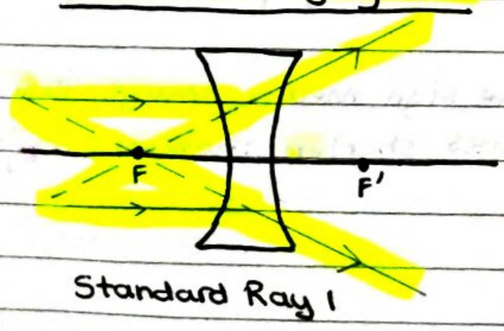
∴ Image is 4 times larger than object & is upright.

Converging lens summary

$u > f$, = real image, inverted
$u < f$, = virtual image, upright
$u = f$, = no image

$u > 2f$, real, inverted, diminished
$f < u < 2f$, real, inverted, magnified

C1.3 Diverging Lens



Standard Ray 1

- Lenses that are thinner at the centre are diverging lenses. Rays change direction away from the axis of lens.
- Rays parallel to pritipal axis are refracted so that they intersect principal axis at focal point F.

- Other parallel rays will come from a point off the principal axis at a distance from the centre of lens equal to focal length. So, at a vertical surface on focal point.

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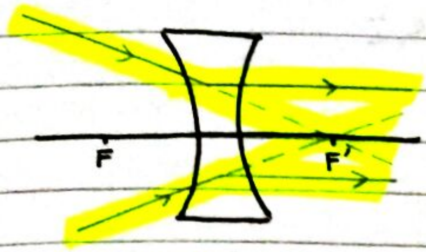


Standard Ray 2

A ray directed towards Focal point on otherside emerges parallel to principal axis.

Standard ray 3

A ray directed at the centre of lens passes through undeflected.

> Exam tip!
> In using thin-lens formula for diverging lens, focal length is negative.

Other conventions are same just like in converging lenses.

- Example. Diverging lens of focal length 10 cm and object placed 15 cm from lens.

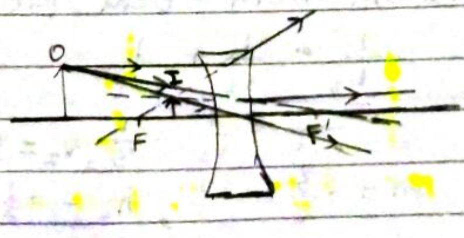
$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{10} - \frac{1}{15} = \frac{-5}{30} = -\frac{1}{6}$$

$$v = -6 \text{ cm}$$

– sign implies image is virtual and on same side as lens of the object.

$$m = -\frac{v}{u} = -\frac{(-6)}{15} = \frac{2}{5}$$, implying an upright image at 40% of height of the object.

> A diverging lens always produces a virtual image ($v < 0$).
> The magnification is then always positive, implying an upright image.



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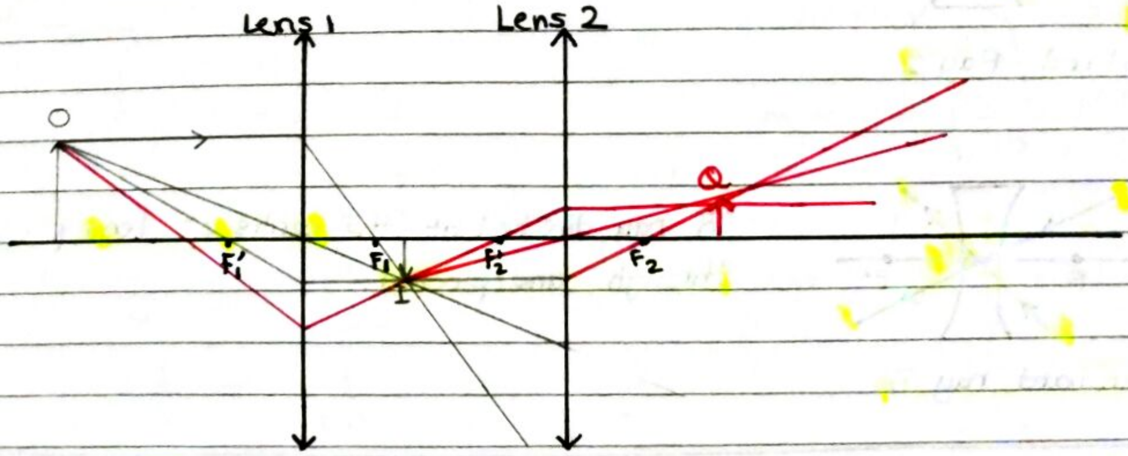
C1.4 Lens combinations : virtual objects

Two Converging Lenses



The pencil ~~mark~~ rays show the 3 standard rays through lens 1. At the second lens, only the ray parallel to principal axis is standard. We need another ray. So, we take a ray from I that passes through the centre of the second lens. We could also draw one that passes through F_2' and gets refracted parallel to principal axis. The diagram is not drawn accurately!

Algebrically:

Example

Two converging lens are 12 cm apart. Left lens f is 4.0 cm and right lens f is 2.0 cm. A 4 cm tall object is placed 12.0 cm to the left of the left lens. What are the characteristics of final image?

$\frac{1}{f_1} = \frac{1}{u_1} + \frac{1}{v_1}$

$\frac{1}{4} = \frac{1}{12} + \frac{1}{v_1}$

$\frac{1}{6} = \frac{1}{v_1}$

$v_1 = 6\text{ cm}$

Using $m = -\frac{v}{u} = \frac{h_i}{h_o}$

$m = -\frac{6}{12} = -\frac{1}{2}$

So, image of lens 1 is 6 cm from lens 1 and is inverted. Since $m = -\frac{1}{2}$, height $-\frac{4}{2} = -2\text{ cm}$. Image is real.

$12\text{ cm} - 6\text{ cm} = 6\text{ cm}$ from Lens 2.

$\frac{1}{f_2} = \frac{1}{u_2} + \frac{1}{v_2}$

$\frac{1}{2} - \frac{1}{6} = \frac{1}{v_2} = \frac{1}{3}$, $v_2 = 3\text{ cm}$ ∴ Real image (+ sign)

$m = -\frac{3}{6} = -\frac{1}{2}$, $h_i = -\frac{1}{2}\times(-2) = 1\text{ cm}$ ∴ Upright

It is diminished to a height of 1 cm. So a factor of $\frac{1}{4}$ from original height ~~of~~ 4 cm.

Exam tip !
Let h_1 be height of original object. h_2 be height of first image and h_3 be height of final image. The overall magnification is :

$$M = \frac{h_3}{h_1} = \frac{h_3}{h_2} \times \frac{h_2}{h_1} = m_1 m_2$$

Example (More interesting)

Two converging lenses are 8.0 cm apart. Focal lengths are 6.0 cm and 4.0 cm for left and right. Object is 12 cm from first lens. Find characteristics of the final image.

$$\frac{1}{f_1} = \frac{1}{u_1} + \frac{1}{v_1}$$

$$\frac{1}{6} = \frac{1}{12} + \frac{1}{v_1}$$

$$\frac{1}{12} = \frac{1}{v_1}, \quad v_1 = 12\text{ cm}$$

$$-\frac{12}{12} = -1 = M$$

Image is just inverted.

But second lens is 8.0 cm from first, so image supposedly was going to form at a point after the second lens. Thus, ~~object~~ image is virtual and this means that u_2 is negative as this virtual image is a virtual object for next lens.

$$\frac{1}{f_2} = \frac{1}{u_2} + \frac{1}{v_2}$$

$$\frac{1}{4} + \frac{1}{(12-8)} = \frac{1}{v_2}$$

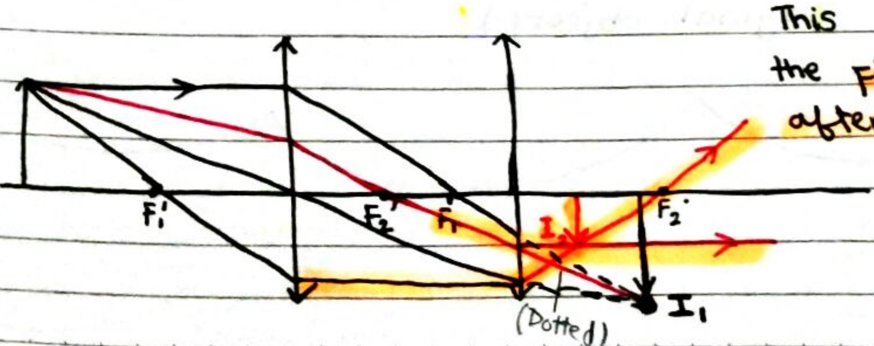
$$\frac{1}{v_2} = \frac{1}{2}, \quad v_2 = 2\text{ cm}$$

$$m_2 = -\frac{2}{-4} = \frac{1}{2}$$

$$m_1 m_2 = -1 \times \frac{1}{2} = -\frac{1}{2}$$

Overall magnification is $-\frac{1}{2}$.

Final image is real as v_2 is positive. It is inverted and has half the height of original object.



This is the illustration where the first image is virtual as it is after the second lens.

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Worked Examples

C.4 An object lies on a table. Converging lens of f 6.0 cm is 4.0 cm above the object.

a) Determine image formed.

b) A second converging lens of focal length 5.0 cm is 3.0 cm above 1st lens. Determine the image formed by this combination of lenses.

a) $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{6} - \frac{1}{4} = \frac{1}{v}$

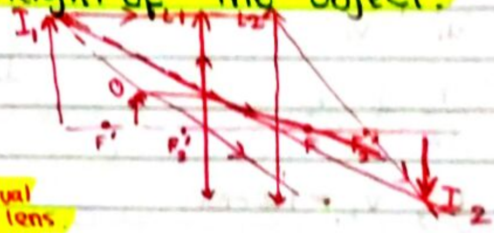
$-\frac{1}{12} = \frac{1}{v}$

$v = -12\text{ cm}$

So, image is virtual. But, not virtual object for next lens. Still on left side.

$m_1 = -\frac{(-12)}{4} = +3.0\text{ cm}$

Image is upright and 3 times the height of the object.



b) (You are thinking that the image should go through lens 1 again. Also that f_2 is going through the first lens.) But, the image only acts as object on lens 2 and it does not matter what the structure is.

$u_2 = 12 + 3 = 15\text{ cm}$

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{5} = \frac{1}{15} + \frac{1}{v}$

$\frac{2}{15} = \frac{1}{v}$

$v = 7.5\text{cm}$

So, final image is real.

$m_2 = -\frac{7.5}{15} = -\frac{1}{2}$

$M = m_1 m_2 = 3 \times -\frac{1}{2} = -\frac{3}{2}$

Overall mag. is $-\frac{3}{2}$.

Final Image is inverted and is 1.5 times taller than original objected.

Worked Examples

C.5 An object is placed 8.0cm to the left of a converging lens of focal length 4.0 cm. A second diverging lens of f 6.0cm is placed 4.0cm to right of the converging lens. Determine image of object in the 2-lens system, and verify your results with a scaled ray diagram.

Solution

$$\frac{1}{4.0} = \frac{1}{8} + \frac{1}{v_1}$$

$$\frac{1}{8} = \frac{1}{v_1}$$

$$v_1 = 8\text{cm}$$

∴ Real Image 1.

$$m_1 = -\frac{8}{8} = -1$$

∴ Inverted and same height.

Image 1 is $(8-4)$cm to the right of diverging lens. So, object 2 is virtual and u is negative, -4cm.

Since diverging lens, f is negative, -6.0cm

$$\frac{1}{f_2} = \frac{1}{u_2} + \frac{1}{v_2}$$

$$\frac{1}{-6} = \frac{1}{-4} + \frac{1}{v_2}$$

$$\frac{1}{4} - \frac{1}{6} = \frac{1}{v_2}$$

$$\frac{1}{12} = \frac{1}{v_2}, \quad v_2 = 12\text{cm}$$

∴ Real image (to the right of both lenses)

$$-\frac{v}{u} = M_2 = \frac{-12}{-4} = +3$$

$$M = +3 \times -1 = -3$$

∴ ~~Upright~~ Inverted final image that is 3 times taller than original object.

$\frac{1}{4} = \frac{1}{8} + \frac{1}{v}$

$\frac{1}{8} = \frac{1}{v}$ $v = 8$ cm

$m_1 = -\frac{8}{8} = -1$

$u_2 = -4$

$\frac{1}{-6} = -\frac{1}{4} + \frac{1}{v}$

$-\frac{1}{6} + \frac{1}{4} = \frac{1}{v_2}$

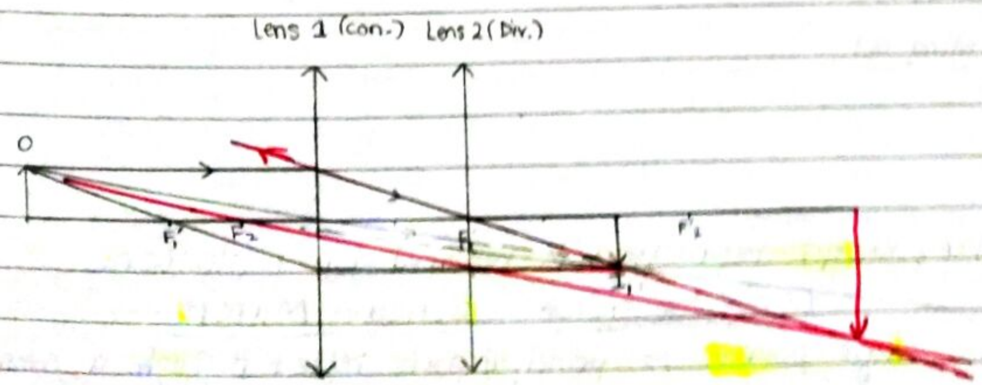
$\frac{1}{12} = \frac{1}{v_2}$

$v_2 = 12$ cm

$-\frac{12}{-4} = 3$

$m_1 m_2 = -3$

(Scale factor $\frac{1}{2}$) (1cm : 2cm)

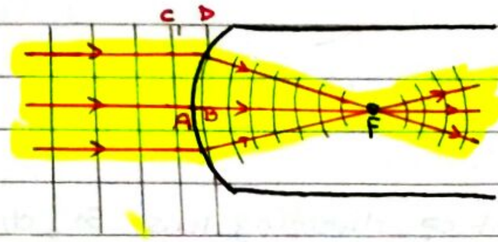


A bit inaccurate!

C1.5 Wavefronts and Lenses

Wavefronts are perpendicular to rays. It is a surface that travels with a wave but is perpendicular to direction of energy propagation.

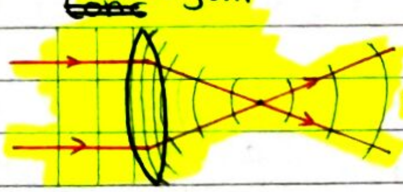
It is easy to draw wavefronts as they are just perpendicular to the rays. Just remember that! Since rays change direction as they get refracted, wavefronts change shape.



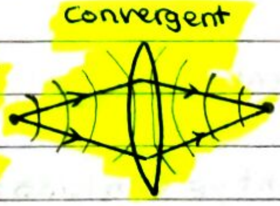
- The wavefronts AC reaches curved boundary at A and gets refracted so that speed is lower and wavelength is lower in this medium. It travels a wavelength λ, which is shorter than other points as they are in air. The wavefront curves. You can see that the wavefront is always ~~perpendicu~~ perpendicular to all rays.

The rays converge towards point F in the medium and from there, rays diverge.

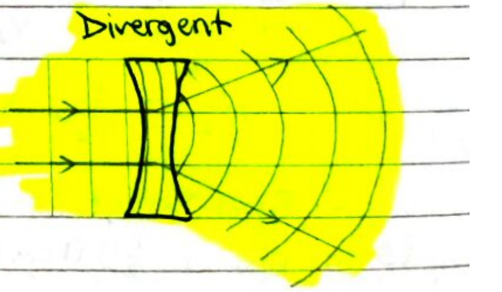
~~Conc~~ Convergent



Convergent



Divergent

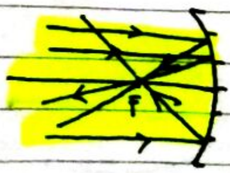


Wavelength gets smaller in lens only. Returns to normal after coming out.

C1.6 Mirrors

Pretty much similar, except refraction is replaced by reflection.

~~Convex mirrors~~ Concave Mirrors



Rays parallel to principal axis reflect through a common point on the principal axis - the focus of the mirror.

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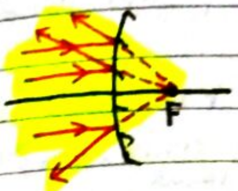
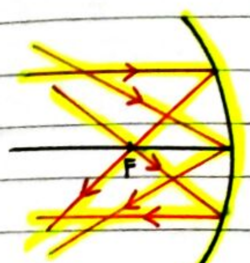


Convex Mirrors

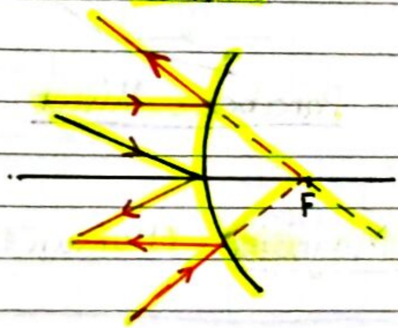
Rays parallel to principal axis reflect so that their extensions go through common point on principal axis, behind mirror - the focus of the convex mirror.

The three standard rays

Concave



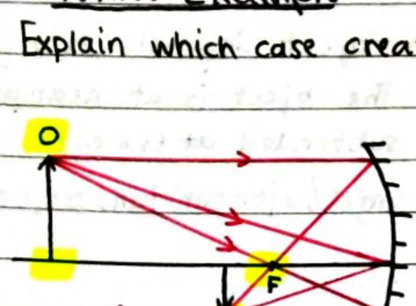
Convex

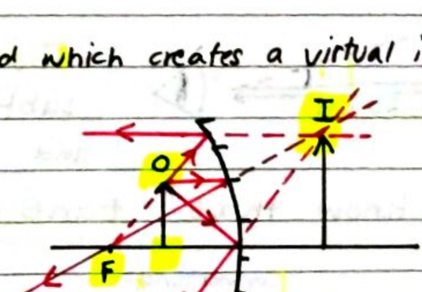


- The Formula about focal length, object distance & image distance also applies to mirrors. Same conventions.
- Magnification formula is also same.
- Convex mirrors (just like divergent lenses) have a negative focal length f.
- v is positive on the left side. Negative on the right side of mirror.

Worked Example

Explain which case creates a real image and which creates a virtual image.





This is a real image because it is formed by real rays.

This is a virtual image because it is formed by ray extensions.

- Rays parallel to principal axis in a spherical mirror only reflect through the same point (Focus) when the rays are very close to the principal axis.

  Rays close to principal axis that pass through the focus as they are parallel to principal axis are called Paraxial rays.

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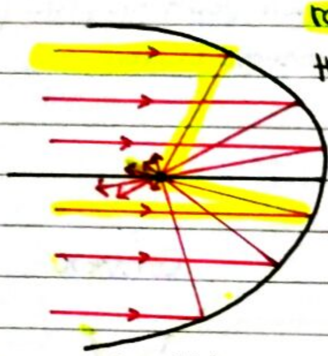
We need a parabolic mirror if we want all rays that are parallel to pass through focus.

It does not matter how far the rays are from principal axis. As long as they are parallel to it, they reflect through focus.

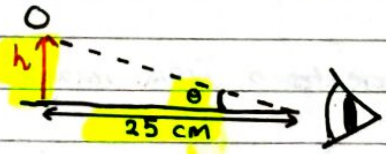


Parabolic Mirror

C1-7 The Magnifier (Human eye)

The closest point to which the human eye can focus without straining is called the near point of the eye. The distance D between near point and human eye is about 25 cm, but it depends greatly on age.

- Between infinite distance to near point distance, the eye can produce a clear, sharp image of any object. But, eyes strain or image is blurred when closer than near point.



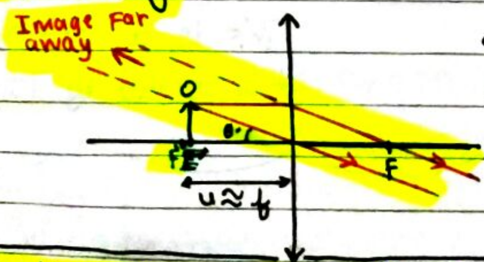
The apparent size of an object depends on angle subtended at the eye. The object is at near point and $D = 25cm$. θ = angle subtended at the eye.

We know that $\tan\theta = \frac{h}{D}$. For very small angles (object far) $\tan\theta \approx \theta$, $\theta = \frac{h}{D}$

Let's say we use converging lens or glasses (with converging lens).

- The object is close to focal point of lens. Between lens & focal point.

$u \approx f$



- A virtual, upright, enlarged image is formed far from lens, which the eye can see comfortably and without straining.

A converging lens acts as a magnifier. The image is formed very far from the lens when the object is just to the right of focal point. Eye then sees it without straining.

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Exam tip!

$\tan\theta = \frac{h}{D}$

If angle is very small,

$\tan\theta \approx \theta$, $\theta = \frac{h}{D}$

Angular Magnification - ratio of the angle subtended ~~by the object when~~ at the eye by the image to the angle subtended by object when viewed by the unaided eye at the near point:

$$M = \frac{\theta'}{\theta}$$

$M = \frac{D}{f}$	Tip! Only when the image is formed far from the lens at infinity.

$D = 25$ cm for human eye.

Because the infinite image meets at this angle. Similar triangles.

Using small-angle approximation, $\theta = \frac{h}{D}$ and $\theta' = \frac{h}{u}$. $u \approx f$.

so $\theta' \approx \frac{h}{f}$

$M \approx \dfrac{\frac{h}{f}}{\frac{h}{D}} \approx \frac{D}{f}$	When image is formed at infinity

~~If not, then $M = \frac{D}{f}$~~

What if we move the object further to the right from F so that it forms image at near point.

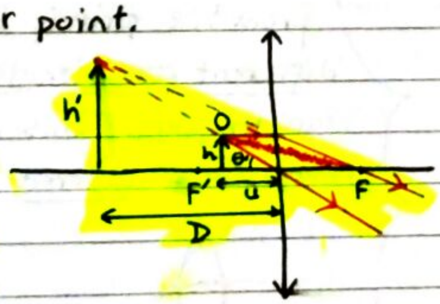

Image is virtual so D is negative.

$$\frac{1}{u} + \frac{1}{-D} = \frac{1}{f}$$

$$= \frac{1}{u} = \frac{f+D}{Df}$$

$$u = \frac{Df}{D+f}$$

Take θ' as the angle subtended by lens at image.

$$\theta' = \frac{h}{u} = \frac{h(D+f)}{D}$$ (No need to ~~dre~~ derive.)

Largest possible M occurs here.

$M = 1 + \frac{D}{f}$	Angular Magnification when image is formed at the near point.

So when image is at 25 cm. $v = D$. This formula applies.

In both cases, decreasing focal length f increases angular magnification.

But lens defects (aberrations) limit M to about 4.

KIKY

Date

Worked Example.

C.7 An object of length 4.0 mm is in front of converging lens of ~~f~~ 6.0 cm. A virtual image is formed 30 cm from lens.

a) Calculate distance of object from lens.
b) Calculate length of image.
c) Calculate angular magnification of lens.

a) $\frac{1}{6} = \frac{1}{-30} + \frac{1}{u}$

$\frac{6}{30} = \frac{1}{u}$

$u = 5\text{ cm}$

b) $-\frac{v}{u} = \frac{30}{5} = 6$

$m = 6$

$6 = \frac{h'}{0.4}$

$h' = 2.4\text{ cm}$

c) $M = \frac{\theta'}{\theta}$

~~$M = \frac{2.4}{30}$~~

$M = \frac{\frac{h}{5}}{\frac{h}{25}} = \frac{25}{5} = 5.0$

$M = \frac{\frac{h}{u}}{\frac{h}{D}} = \frac{D}{u}$

The D for human eye.

C1.8 Lens Aberrations

Lens and Mirrors suffer from aberrations.

~~Aberrations = deviations from the simple description of~~

They are two types:

1) Spherical aberration

When rays that enter the lens far from the principal axis have a slightly different focal length from rays entering near the axis. This means that the image is ~~a~~ blurred ~~patch of light~~ and curved.

Solution - reducing aperture of lens (its diameter). This is called stopping down.

However, stopping down means less light goes through the lens and so the image is less bright. The light ray will also undergo more pronounced diffraction.

Mirrors suffer just like lenses. The magnification varies from ray to ray and this leads to image distortion.

Basically stopping down means lens or mirror becomes from .

2) ~~Chromatic aberration~~ - When lens have different refractive indexes for different wavelengths, there is a separate focal length for each λ of light.

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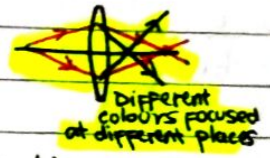
2) **Chromatic aberration** See aberration question 23 ~~beta~~ in TYQ.

When lens have different refractive indices for different wavelengths,
∴ there is a separate focal length for each wavelength (colour) of light.

Images appear ~~faintly coloured~~ blurred and coloured faintness - there are lines around the image in the colours of the rainbow. (Can imagine a bad print)



Solutions
- Use monochromatic light.
- Combining lenses with different n to reduce aberration.

MIRRORS DO NOT SUFFER FROM CHROMATIC ABERRATION.

Test Yourself (Important questions that are new)

1 a) Define focal point of a converging lens - the point on the principal axis where rays parallel to the principle axis will refract through after going through the lens.

b) Define focal length of a diverging lens - the distance between the focal point and centre of the diverging lens. It is taken negative for diverging lens.

---

2 a) Real image : Formed by actual rays and can be projected on screen.

b) Virtual image : Formed by extension of rays and cannot be projected on screen.

---

3) Why a real image can be projected on a screen but a virtual image cannot?

Ans. The actual rays of light that can go through the image will reflect off the screen and so it can be seen. With virtual images, placing a screen in front of image reveals nothing as there are no rays of light to reflect off the screen.

$u+v=500$ $\frac{1}{60}=\frac{1}{u}$ $60 \leftarrow u$

15) An object is 5.0 m from screen. Converging lens of f 60 cm is placed between object and screen so that image of object is formed on screen. a) Determine distances from screen ~~for which lens~~ where lens could be placed. b) Determine which choice leads to larger image.

a) Image must be real. So v is positive. $\frac{1}{60}=\frac{1}{u}+\frac{1}{v}$, $u > 60$ for v to be positive.

~~500 - 60 = 440 cm Lens can be placed at any distance less than 440 cm~~ $u+v=500$ cm

~~b) $m=-\frac{v}{u}$, v is largest~~ $u = 500 - v$; $\frac{1}{60}=\frac{1}{500-v}+\frac{1}{v}=\frac{500}{500v-v^2}$; (Solve) $v = 430$ cm or 70 cm

b) $m=-\frac{v}{u}$, so image is larger when $v = 430$ cm as m is larger.

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Date

17) Two very thin lenses of f_1 and f_2 are in contact. Show that focal length of the 2-lens system is $f = \frac{f_1 f_2}{f_1 + f_2}$

For object incident at first lens:

$$\frac{1}{f_1} = \frac{1}{u_1} + \frac{1}{v_1}$$

$$\frac{1}{v_1} = \frac{1}{u_1} - \frac{1}{f_1}$$

The image is virtual for f_2. ~~regardless of where it forms. (Think about it)~~

$$\left(-\frac{1}{v_1}\right) + \frac{1}{v_2} = \frac{1}{f_2}$$

$$\frac{1}{u_1} - \frac{1}{f_1} + \frac{1}{v_2} = \frac{1}{f_2}$$

$$\frac{1}{u_1} + \frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{f_1}$$

$$\frac{1}{v_2} = \left(\frac{1}{f_1} + \frac{1}{f_2}\right) - \frac{1}{u_1}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{f_2 + f_1}{f_1 f_2}$$

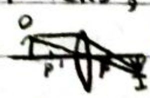
$$f = \frac{f_1 f_2}{f_1 + f_2}$$

When 2 con. lenses are close.

18) Two converging lenses, each of focal length 10.0 cm, are 4.00 cm apart. Find focal length of this lens combination.

For object rays incident on 1st lens,

$$\frac{1}{10} = \frac{1}{u_1} + \frac{1}{v_1}$$



$$\frac{1}{10} - \frac{1}{u_1} = \frac{1}{v_1}$$ (Convex lens image always forms after F)

So, image, is on right side of lens 2.

$$\frac{1}{10} = -\frac{1}{v_1} + \frac{1}{v_2} = -\left(\frac{1}{10} - \frac{1}{u_1}\right) + \frac{1}{v_2}$$

$$\frac{2}{10} = \frac{1}{v_2} - \frac{1}{u_1}$$ $f = 5$ cm

$$\frac{1}{5} = \frac{1}{v_2} - \frac{1}{u_1}$$

OR $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$\frac{1}{f} = \frac{1}{10} + \frac{1}{10}$

$f = 5$ cm

$f = \frac{f_1 f_2}{f_1 + f_2}$

$= \frac{10 \times 10}{20}$

$= 5$ cm

(KIKY)

Date

21) An object is viewed through a system of 2 lenses L_1 and L_2. L_1 is convergent ~~at~~ with f 35.0 cm and L_2 is divergent with f 20. cm. Distance between lenses is 25.0 cm and distance between object and L_1 is 30.0 cm. Determine:

a) Position of image
b) Magnification of image
c) Orientation of image

a)
$$\frac{1}{35} = \frac{1}{30} + \frac{1}{v}$$
$$\frac{6-7}{210} = \frac{1}{v}$$
$$v = -210 \text{ cm}$$
Virtual image to the left of lenses.

~~$\frac{1}{-20} = \frac{1}{210-25} + \frac{1}{v}$~~ But it is real object for lens 2 as it is on left.

~~$\frac{1}{-20} = \frac{1}{235} + \frac{1}{v}$~~

$$\frac{1}{-20} = \frac{1}{210+25} + \frac{1}{v}$$

~~$-\frac{43}{940} = \frac{1}{v}$~~

$$v = -18.4 \text{ cm}$$

~~$v = -21.9$ cm~~

~~∴ Final Image is 21.9 cm from lens 2 to the left.~~ ∴ Final image is 18.4 cm to the left of L2.

b) $M = M_1 M_2$

$$M_1 = \frac{+210}{30} = 7$$

$$M_2 = +\frac{18.4}{235} = 0.078$$

$$M = 7 \times 0.078 = 0.55 \text{ or } 0.548$$

c) Image is upright and 0.548 times the height of original object.

22) An object 15 mm high is 12 cm from a mirror. An upright image 30 mm high is formed. Determine focal length of mirror and whether it is concave or convex.

$$\frac{h'}{h} = -\frac{v}{u}$$

$$\frac{3.0}{1.5} = -\frac{v}{12}$$

$$-24 = v = -24 \text{ cm}$$

So, virtual image to the right of mirror.

$$\frac{1}{f} = \frac{1}{12} + \frac{1}{(-24)}$$

$$\frac{1}{f} = \frac{1}{24}$$

$$f = 24 \text{ cm}$$

f is positive, so mirror is concave.

Date

23 a) Describe the two main lens aberrations and indicate how these can be corrected.

Spherical aberration occurs because light rays at a higher distance from the principal axis refract at a different focus point on p-axis then ray at a lower vertical distance from the principal axis. These cause image to be blurred and curved, but can be corrected by reducing the diameter of the lens, also called stopping down. This only allows rays close to principal axis to be refracted.

Chromatic aberration occurs in lenses when the lens refracts different wavelengths or colours of light at different focal points on the principal axis, causing blurriness & color faintness of image. This is due to different refractive indices of different colours. It can be reduced by combining lenses with different refractive index. Like a convergent lens with a divergent lens.

24) An object at converging lens, real image formed on other side. Distance of object from left focal point is x and distance of image from right focal point is y. Show that $xy = f^2$.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$u = f + x$ $\quad$ $v = f + y$ (Think about it! $u > f$ for real image and $v > f$ always at real image)

$$\frac{1}{f} = \frac{1}{f+x} + \frac{1}{f+y}$$

$u = x + f$

$v = y + f$

$\frac{1}{f} = \frac{1}{x+f} + \frac{1}{y+f}$

$xy = f^2$

$$\frac{1}{f} = \frac{2f + x + y}{(f+x)(f+y)}$$

$(f+x)(f+y) = 2f^2 + fx + fy$

$f^2 + fy + fx + xy = 2f^2 + fx + fy$

$xy = f^2$

27) Human eye distinguishes 2 objects 0.12 mm apart when they placed at near point. A simple magnifying glass of f 5.00 cm views images at near point. Determine how close the objects can be and still be distinguished.

$M = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 6.0$.

Now it is easy to distinguish them at smaller distance since angularly magnified.

$\frac{0.12}{6} = 0.02\ mm = 0.00002 = 2 \times 10^{-5}\ M$

(KIKY)

$\frac{d_1}{M} = d_2$

Date

C2 Imaging Instrumentation

L.O.

- Compound microscopes
- Astronomical refracting and reflecting telescopes
- Single-dish radio telescopes (outline its use)
- Radio interferometry telescopes (understand principal)
- Satellite-borne telescopes (Advantages)

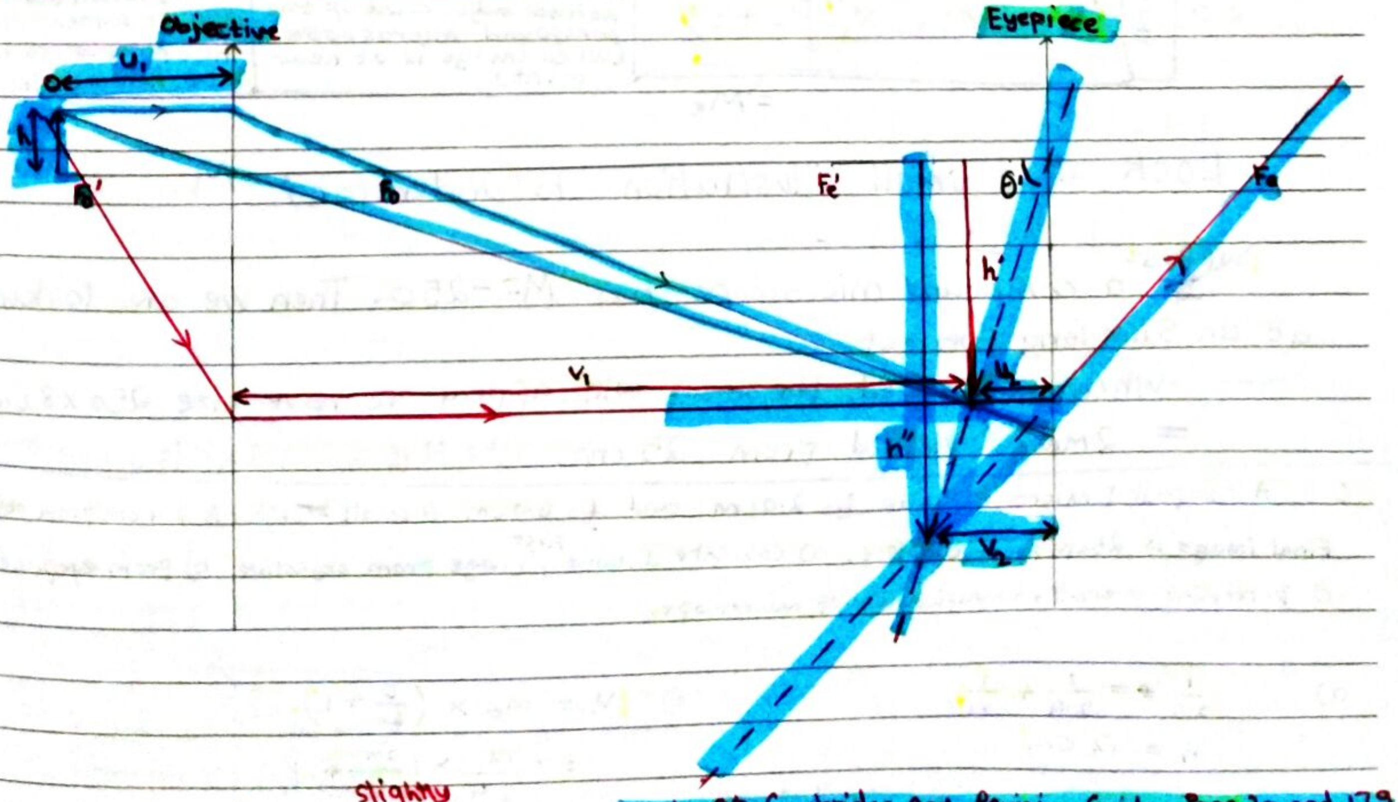
C2.1 The Optical Compound Microscope (Learn diagram)

M is always negative.
v_2 is always negative.

A compound microscope consists of 2 ~~diverging~~ converging lenses.

Used to see enlarged images of very small objects.

1) Object of height h is placed at distance slightly greater than focal length f_o of the objective lens.
2) Real inverted image of height h' formed in front of second lens (eyepiece) to the left and to the right of its focal point Fe.
3) The image is an object for eyepiece, and produces an enlarged, virtual image of h'' as a magnifier.



This image is slightly inaccurate. Look at Cambridge and Revision Guide. Page 20 and 179.

$M = M_e \times m_o = \left(\frac{D}{f_e}+1\right) \times m_o = \left(\frac{D}{f_e}+1\right) \times \left(-\frac{v_1}{u_1}\right) = \left(\frac{D}{f_e}+1\right) \times \left(1 - \frac{v_1}{f_o}\right)$

Microscope:

Always at normal adjustment for compound microscope.

Question should mention "normal adjustment" or not.

If ~~not~~ at infinity, ~~normal adjustment~~ $M = -\frac{v_1}{u_1} \times \left(\frac{D}{f_e}\right)$

Angular Magnification $= \frac{\theta'}{\theta}$ — Find $\left(\frac{D}{f_e}+1\right)$ & $\left(\frac{D}{f_e}\right)$ in D.B.

θ' is the angle that final image subtends at eyepiece and θ is angle to which original object would subtend when view at near point D.

$\theta' = \frac{h''}{v_2}$ and $\theta = \frac{h}{D}$, so

Basically, $M = m_o \times M_e$

$M_e = \left(\frac{D}{f_e}\right)$ or $\left(\frac{D}{f_e}+1\right)$

$$M = \frac{\frac{h''}{v_2}}{\frac{h}{D}} \approx \frac{D}{f} = \frac{Dh''}{v_2 h} = \frac{h''}{h'} \times \frac{h'}{h} \times \frac{D}{v_2}$$

These are linear magnifications of objective & eyepiece.

If final image not at near point, use it!

so, ① $M = m_o \times m_e \times \frac{D}{v_2}$, $D = v_2$, so when at near point

Another formula: $M = M_o \times M_e$; $M = \frac{h_2}{h_1} \cdot \frac{h_1}{h}$

$M = m_o \times M_e$

~~$M = -\frac{v_1}{u_1} \times \left(\frac{D}{f_e}\right)$~~

$m_o = -\frac{v_1}{u_1}$, $m_e = -\frac{v_2}{u_2}$

If final image is formed at near point (normal adjustment), then $v_2 = D$ (makes sense) and in that case (remember?) $m_e = \frac{D}{f_e} + 1$, ($M_e = m_e$ here)

so, ② $M = m_o \times \left(\frac{D}{f_e}+1\right)$ ($= M_e$)

Angular magnification at normal adjustment of the compound microscope. (when image is at near point)

Here, linear & angular mag. is same of eyepiece. These formulas are not in DB. Remember them!

Look at a small illustration to understand M.

Suppose a compound microscope has $M = -250$. Then we are looking at an 8µm long object.

When magnified, the object will appear to have size 250×8µm $= 2$mm, viewed from 25 cm. The M of a microscope is always negative.

C.8 A compound microscope has f_o 2.0cm and f_e 6.0cm. A small object is 2.4 cm from objective. Final image is 25cm from eyepiece. a) Calculate distance of first image from objective b) From eyepiece c) Determine overall magnification of microscope.

a) $\frac{1}{2.0} = \frac{1}{2.4} + \frac{1}{v_1}$

$v_1 = 12$ cm

b) $\frac{1}{6} = \frac{1}{u_2} + \frac{1}{(-25)}$ — We know that the v_e is always negative.

$u_2 = 4.8$ cm

c) $M = m_o \times \left(\frac{D}{f_e}+1\right)$

$= -\frac{12}{2.4} \times \left(\frac{24}{6}+1\right)$

$M = -5 \times 5 = -25$

(Just memorize $M = m_o \times \left(\frac{D}{f_e}+1\right)$)

KIKY

In conclusion, Final image is inverted & virtual. ~~[struck out]~~ M is negative and v_e is negative.

For Final image at normal adjustment, $M = m_o \times M_e = -\frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$. $m_e = M_e$

For Final image at infinity, $M = m_o \times M_e = -\frac{v_o}{u_o}\left(\frac{D}{f_e}\right)$

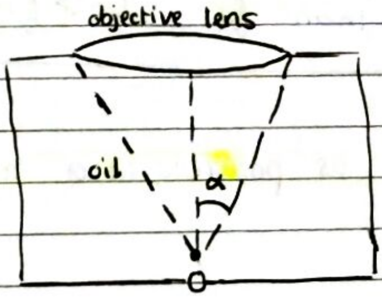
Date

C 2.2 Resolution of a compound microscope

- If 2 point sources are very close to each other, their images will overlap and so may not be seen as distinct. This limits the resolution of the compound microscope.

The smallest distance that can be resolved in a microscope is $d_{min} = \frac{0.61\lambda}{n \sin\alpha}$ λ = wavelength of the light; n = refractive index of oil (read below); α = angle (shown below)	Not needed for the exam.

We are using an oil immersion ~~telescope~~ microscope to reduce d_{min} to increase resolution.



Not really needed for the exam. Might come out?

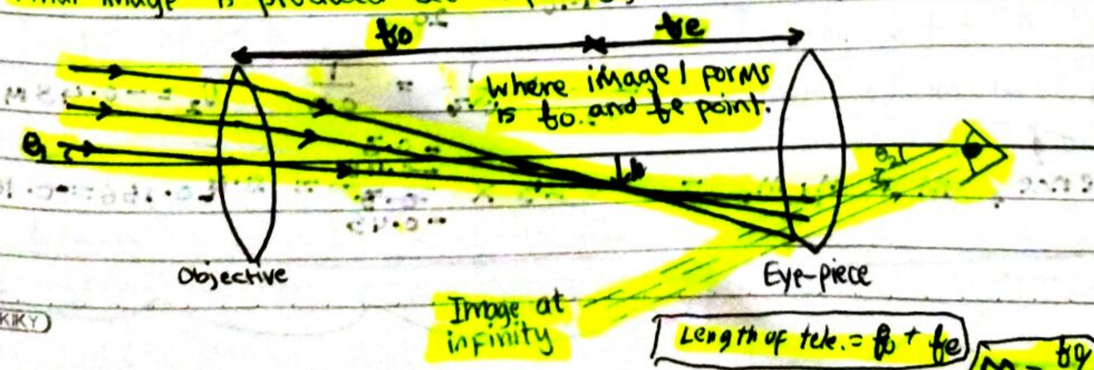
C 2.3 The refracting telescope (Learn diagram)

- Function of a telescope – allow observation of large objects that are very distant and appear small.

- The telescope increases angle subtended by the star relative to the angle subtended at the unaided eye.
  TELESCOPE DOES NOT PROVIDE LINEAR MAGNIFICATION.

- Refracting telescope has 2 converging lenses.

1. Objective lens produces a real but diminishes image of the distant object.
2. The image produced by objective lens is magnified into a virtual, inverted image by the eyepiece.
3. Final image is produced at infinity, and telescope is under normal adjustment.



IMPORTANT! Length of telescope = $f_o + f_e$

$M = \frac{\theta_2}{\theta_1} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}$

Length of tele. = $f_o + f_e$

IMPORTANT! $M = f_o/f_e$

Date

Again! **Length of telescope = $f_o + f_e$**

$$M = \frac{f_o}{f_e}$$

Formula for angular magnification of a refracting telescope with image at infinity (normal adjustment)

Huge thing to know:
Microscope has smaller f_o than f_e. $f_o < f_e$
Telescopes have larger f_o than f_e. $f_o > f_e$

- The objective lens is as large as possible in a refracting telescope to allow more light into it.
- Disadvantage – It is hard to make these telescopes as they need very large ~~refracting~~ objective lenses. So, mirror telescopes are built.

Worked Examples

C.10 A refracting telescope has a magnification 70.0 and the two lenses are 60.0 cm apart at normal adjustment. Determine focal lengths of the lenses.

Simultaneous or substitution

$60.0 = f_o + f_e$

$70.0 = \frac{f_o}{f_e}$

$70 f_e = f_o$

$60.0 = 70 f_e + f_e$

$\frac{60}{71} = f_e = 0.845\ \text{cm}$

$f_o = 59.2\ \text{cm}$

(Recheck by ensuring $f_o > f_e$.)

C.11 An astronomical telescope is used to view an object 20 m from ~~objective~~ objective. Final final image is 30 cm from ~~objective~~ eyepiece lens. f_o is 4.0 m and f_e is 0.80 m. Determine overall linear magnification of telescope.

$\frac{1}{20} + \frac{1}{v_1} = \frac{1}{4}$, $v_1 = 5\ \text{m}$

$M_0 = -\frac{5}{20} = -0.25$

$\frac{1}{u_2} + \frac{1}{0.3} = \frac{1}{0.8}$ $u_2 = -0.48\ \text{m}$

Hence, $M = M_1 M_2 = -0.25 \times \frac{-0.3}{-0.48} = -0.156 = -0.16$

Date

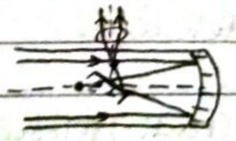
C2.4 Reflecting Telescopes

Comparison of reflecting and refracting telescopes

Refracting T.	Reflecting T.
Theory : Uses an objective (converging) lens to form a real, diminished image of a distant object. This image is viewed by eyepiece (converging), which acts as a magnifying glass and produces a virtual, magnified image (final).	Theory : Uses concave Mirror to form a real, diminished image of an object. Once again, the eyepiece is a converging magnifying lens and produces the virtual, final image. Two types : Newtonion Mounting, Cassegrain Mounting

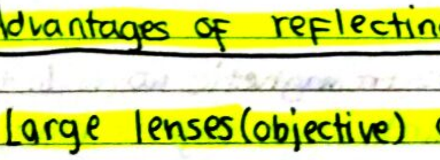
Advantages of reflecting telescopes

1. Large lenses (objective) are needed to collect more light, but these are difficult to make (should be free of air bubbles). They also collapse under their own weight. However, large mirrors can be supported from below.
2. Mirrors do not suffer from chromatic aberration.
3. ONLY one side has to be right shape, as opposed to two for lenses.
4. It is difficult to get a uniform refractive index throughout a large lens.



Disadvantages of reflecting telescopes

1. The mirror surface can be easily damaged.



Newtonian telescope

45° Newtonian 45 Telescope

Light from a distant object is reflected by concave parabolic mirror onto small plane mirror at 45° to axis of telescope. The reflected light is collected by converging lens that creates a parallel beam to the observer's eye.

Cassegrain telescope

Light is reflected from a concave parabolic Mirror onto a much smaller convex mirror. Light reflecting off this mirror is collected by a converging lens that produces a parallel beam to observer's eye.

C. 2.5 **Single-dish radio telescopes**

A radio telescope receives and detects electromagnetic waves in the radio-frequency region. Stars, galaxies, and other objects radiate in this region, so we get information about them.

Remember from 9.4, an instrument with diameter b that operates with λ can resolve two objects with angular separation $\theta_A \geq 1.22 \frac{\lambda}{b}$.

(λ)

Since, radio wavelengths are large, diameter (b) must be large of the radio telescope.

Arecibo (radio) telescope

$b = 300\,m$ Operates at $\lambda = 21cm$

It can resolve objects with angular separation more than or equal to

$$\theta_A = \frac{1.22 \times 0.21}{300} = 8.5 \times 10^{-4} \text{ rad}$$

Hubble Space (Optical) Telescope

$b = 2.4m$ $\lambda = 500nm$

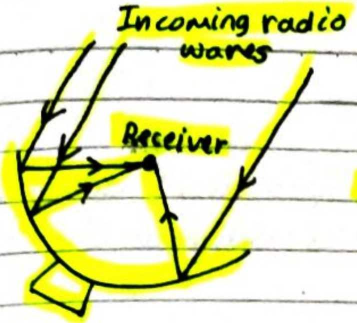
$$\theta_A \geq \frac{1.22 \times 500 \times 10^{-9}}{2.4}$$

$$\theta_A \geq 2.5 \times 10^{-7} \text{ rad}$$

can resolve better.

Disadvantage: Large and difficult to steer. Diffraction limits accuracy of locating individual sources.

Solution: Increasing diameter (b) improves resolution and limits diffraction effects. Also ensures that more power can be received.



Date

1) Incoming radio waves are reflected by dish to the receiver (antenna).
2) Receiver is tuned to pick up specific wavelengths (λ).

C2.6 Radio - Interperometry telescopes

- This is an improvement in resolution of radio telescopes using interferometry.

1) A large array of radio telescopes very far apart are set but they point in the same direction.
2) Through this, we get the same resolution as a single-dish with a diameter equal to the length of the array. So, we have a much larger virtual telescope.

E.g. VLA inferometer has 27 single dishes extending 35 km and operating a wavelength 6 cm.

$$\theta_A = \frac{1.22 \times 0.06}{35000} = 2.1 \times 10^{-6} \text{ rad}$$

This is only 10 times lower the Hubble Space Telescope.

Disadvantage
This process of radio interferometry is complex as each antenna must be carefully calibrated.

(EB) (SB)

C2.7 COMPARATIVE PERFORMANCE OF EARTH-BOUND AND SATELLITE-BORNE TELESCOPES

+1) Space-borne telescopes are free from interference and absorptions in Earth's atmosphere, giving better resolution for SB telescopes.
-2) Modern computer techniques can correct many atmospheric effects though.
+3) SB telescopes do not suffer from light and radio interference due to human activity.
+4) SB telescopes are less likely to depreciate as quickly as EB telescopes. (Storms)
-5) SB telescopes can be damaged by space debris. (collision)
-6) SB telescopes are more expensive. (Launch into space and monitor).
-7) SB telescopes must withstand wider temperature variations than EB telescopes.
+8) EB telescopes can only operate at night unlike SB telescopes.

Date

Nature of Science

Placing telescopes away from Earth has helped avoid distorting effects of Earth's atmosphere and corrective optics enhance Earth-bound observations.

Optical, electron and tunelling microscopes have advanced our knowledge of the biological world, leading to spectacular advances in medicine and treatment of disease.

Test yourself

(Normal Adjustment question)

28) Objective of a microscope has f_o 0.80cm and eyepiece has f_e 4.0cm. An object is 1.50 cm from objective. Final image is at near point of the eye. (25 cm)

a) Calculate distance of image from objective.

$$\frac{1}{f_o} = \frac{1}{u_1} + \frac{1}{v_1}$$

$$\frac{1}{0.80} = \frac{1}{1.50} + \frac{1}{v_1}$$

$$\frac{15-8}{12} = \frac{1}{v_1} = \frac{7}{12}$$

$$v_1 = \frac{12}{7} = 1.71\text{ cm}$$

b) Calculate distance from eyepiece lens of first image.

$$\frac{1}{u_2} + \frac{1}{-v_2} = \frac{1}{4.0}$$

($-v_2$) always negative for microscope.

$$\frac{1}{u_2} = \frac{1}{4} + \frac{1}{25} = \frac{29}{100}$$

$$u_2 = \frac{100}{29} = 3.45\text{ cm}$$

c) Calculate angular magnification of microscope.

$$M = M_1 \times \left(\frac{D}{f_e} + 1\right)$$

$$= \frac{-1.71}{1.50} \times \left(\frac{25}{4.0} + 1\right)$$

$$= -1.14 \times 7.25 = -8.265$$

$$= -8.27 = -8.3 \text{ (2 s.f.)}$$

29) In a compound microscope the objective focal length is 20 mm and eyepiece focal length is 80 mm. An object is 25 mm from objective and the final virtual image is 35 cm from objective.

a) Calculate angular magnification of microscope. ~~(No normal adj)~~ (No near point)

$$\frac{1}{2.5} + \frac{1}{v_1} = \frac{1}{2}, \quad \frac{1}{v_1} = \frac{1}{10}, \quad v_1 = 10\text{ cm}$$

$$\frac{1}{u_2} + \frac{1}{-35} = \frac{1}{8.0}$$

$$\frac{1}{u_2} = \frac{43}{280}$$

$$u_2 = 6.5\text{ cm}$$

$$M = m_1 \times m_2 \times \frac{D}{v_2} \quad \text{when } v_2 \neq D$$

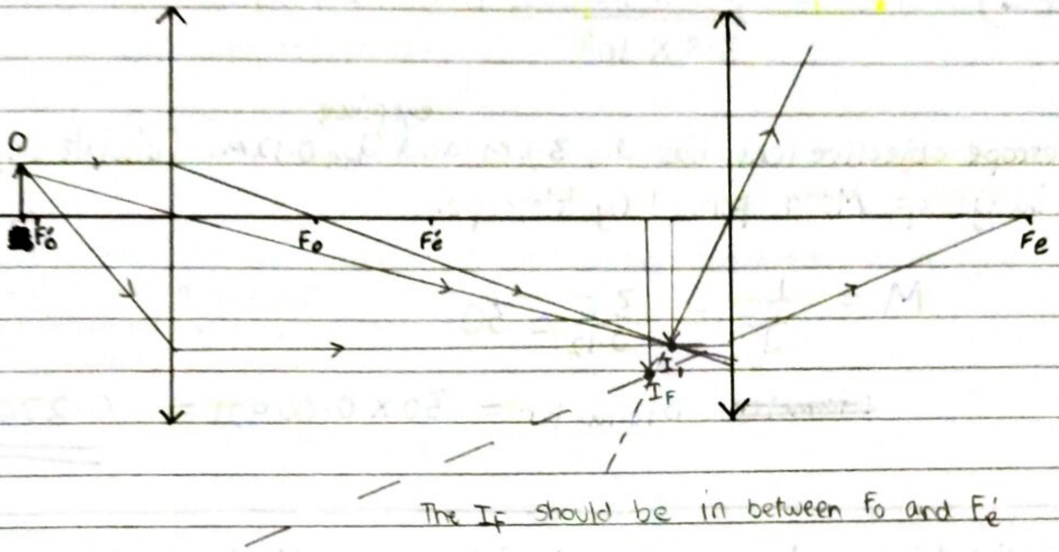
$$M = -\frac{10}{2.5} \times \frac{+35}{6.5} \times \frac{25}{35} = -15.4 \approx -15$$

~~$M = \frac{-10}{2.5} \times \left(\frac{25}{8}\right)$~~

Not at infinity nor normal adjustment

Date

30) Diagram below illustrates a compound microscope. Copy diagram and draw rays to construct final image.



The I_F should be in between F_o and F_e'

31) Compound ~~telescope~~ microscope forms final image 25cm from eye-piece. Eye is very close to eyepiece. f_o is 24mm and object is 30mm from objective. Angular mag. is 30. Determine focal length of the eyepiece.

$$\frac{1}{v_1} = \frac{1}{2.4} - \frac{1}{3.0}$$

$$\frac{1}{v_1} = \frac{1}{12}, \quad v_1 = 12\text{ cm}$$

$$m_1 = -\frac{12}{3} = -4$$

$$M = m_1 \times \left(\frac{D}{f_e} + 1\right)$$

Take M as negative.

$$-30 = -4 \times \left(\frac{25}{f_e} + 1\right)$$

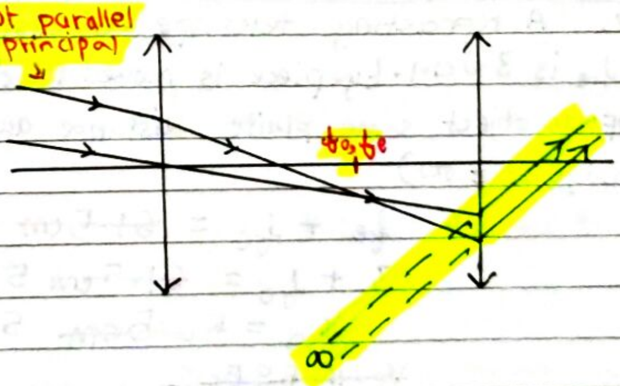
$$+7.5 = \frac{25 + f_e}{f_e}$$

$$+6.5 f_e = 25$$

$$f_e = 3.85$$

$$f_e \approx 3.9$$

32) Complete diagram for refracting telescope where final image forms at infinity.



33) An astronomical telescope is in normal adjustment.

a) State what this statement means.

The final image is at ~~a distance~~ infinity.

b) Angular mag. is 14 and f_o is 2.0m. Calculate focal length of eye-piece.

$$14 = \frac{f_o}{f_e}$$

$$M = \frac{f_o}{f_e}$$

$$14 f_e = 2$$

$$f_e = 0.14\text{ m}$$

Date

34. Moon at distance 3.8×10^8 m from Earth and diameter is 3.5×10^6 m.

a) Show that angle subtended by diameter of Moon at eye of an Earthian is 0.00092 rad.

$$\theta = \frac{3.5 \times 10^6}{3.8 \times 10^8} = 0.0092 \text{ rad}$$

b) Telescope objective lens has f_o 3.6 m and eyepiece f_e 0.12 m. Calculate angular diameter of image of Moon formed by telescope.

$$M = \frac{f_o}{f_e} = \frac{3.6}{0.12} = 30$$

So for observed θ, telescope makes it bigger. For min. $\theta = \frac{1.22\lambda}{b}$, telescope makes it smaller.

~~Diameter~~ Diameter = 30×0.00921 = 0.276 ≈ 0.28

---

38. Refractive telescope has eyepiece f_e 3.0 cm and objective f_o 67.0 cm.

(a) Calculate magnification of telescope. (b) State length of telescope. (Final image is produced at ∞)

$$M = \frac{f_o}{f_e} = \frac{67.0}{3.0} = 22$$

$f_o + f_e$ = 70 cm = length

---

37. A refracting telescope has distance 60 cm between objective & eyepiece. f_e is 3.0 cm. Eyepiece is moved 1.5 cm further from objective to provide clear image of an object some finite distance away. Estimate this distance. (Assume final image at ∞)

$f_e + f_o$ = ~~61.5 cm~~ 60

$3 + f_o$ = ~~61.5 cm~~ 57 cm

f_o = ~~58.5 cm~~ 57 cm

~~v_1 is 58.5 cm~~

61.5 − 3 = 58.5 cm = v_1

Distance − f_e = v_1

$$\frac{1}{u_1} + \frac{1}{58.5} = \frac{1}{57}$$

u_1 = 22 m

KIKY

Date

38. a) What is meant by radio interferometry.

The technique in radio astronomy where an array of radio telescopes observe and detect radio waves and combine them at a single point.

b) Estimate resolution in radians of an array of radio telescopes extending 25 km. λ operated is 21 cm.

$$\theta = \frac{1.22 \times 0.21}{25000} = 1.0 \times 10^{-5} \text{ rad}$$

c) Estimate smallest separation that can be resolved, in a galaxy 2×10^{22} m from the Earth.

$$\theta = \frac{\text{Separation}}{\text{Distance}} = \frac{x}{2 \times 10^{22}}$$

$$x = 2 \times 10^{22} \times 1.0 \times 10^{-5} = 2 \times 10^{17} \text{ m}$$

Descriptive Questions (Important)

39. Why are telescopes other than optical ones being used?

There are sources in the universe emitting all forms of electromagnetic radiation, not just optical light.

40. Why are parabolic mirrors used in telescopes.

They do not suffer from spherical aberration when parabolic as each ray is reflected through one ~~focus~~ focal point.

41. State 2 advantages and 2 disadvantages of satellite-based telescopes.

Advantages

+ They can operate throughout the day while Earth-bound telescope only can operate at night.
+ They do not undergo interference and wavelength absorptions due to atmosphere.
+ Free from atmospheric and light turbulence.

Disadvantages

- Risk of damage due to space debris.
- They must withstand various variations ~~of~~ in temperature.
- Expensive to launch into space.
- Expensive to repair.

KIKY

C3 Fibre Optics

Date

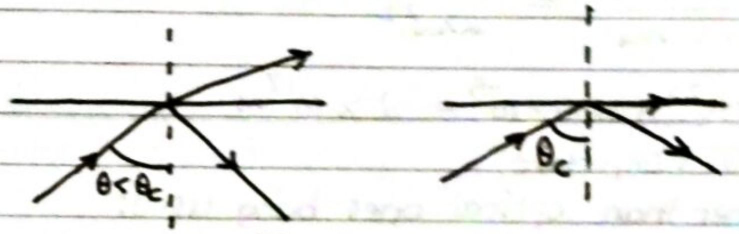
Learning Objectives
- Optic fibres with total internal ref.
- Step-index & graded-index fibres.
- Waveguide and material dispersion of optic fibres
- Attenuation & dB scale
- How waveguide & dispersion lead to attenuation
- Advantages of optic fibres over other cables
- Waveguide dispersion is aka modal dispersion

An important channel of communication.

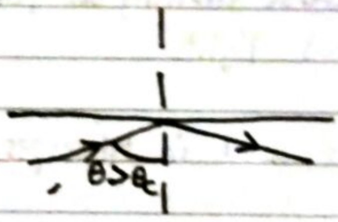
We introduce multimode and monomode fibres, and discuss them in context of dispersion and attenuation.

C3.1 Total Internal Reflection & Optical Fibres

When a ray of light travels from an optically denser medium to an optically less dense medium, the angle of incidence at which the angle of refraction is 90° is called the critical angle.



At any angle above critical angle, there is total internal reflection.



> The critical angle is the angle of incidence for which the angle of refraction is 90°.

Snell's Law : $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$n_1 \sin\theta_1 = n_2 \sin 90°$$

$$\sin c = \frac{n_2}{n_1}$$

$$c = \arcsin\left(\frac{n_2}{n_1}\right) \quad \text{When } n_2 = 1,\ \sin c = \frac{1}{n_1}$$

Application

1) In communication industry, digital data is transferred by light pulses that can travel along the fibre.
2) In medical world, optical fibres carry images back from the patient's body. The instrument is called endoscope.

→ This type of optic fibre is a step-index optic fibre. (Discussed later) Cladding of another material with lower refractive index surrounds the wire, which protects and strengthens the fibre.

(KIKY)

Date

Worked Example

C.12 Refractive index of core of an optical fibre is 1.50 and of cladding is 1.40. Calculate critical angle at core-cladding boundary.

n_1 is optically denser than n_2.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1 \sin c = n_2 \sin 90$$

$$\sin c = \frac{1.40}{1.50}$$

$$c = \arcsin\left(\frac{1.40}{1.50}\right)$$

$$c = 1.20 \text{ rad or } 69.0^\circ$$

Now about optic fibres ...

So, a ray of light travels along a transparent fibre by bouncing between the walls of the fibre.

- As long as the incident angle on the ray on the wall is greater than critical angle, ray remains inside the fibre.

TYPES OF OPTIC FIBRES

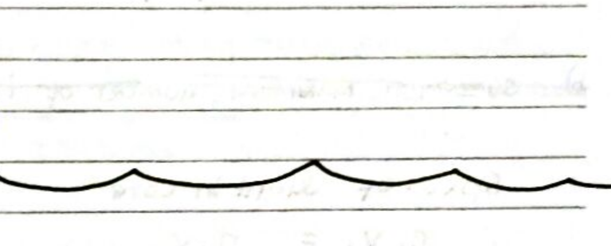
~~Wider outpulse output p~~

Wider output pulse = dispersion

Shorter output pulse = attenuation

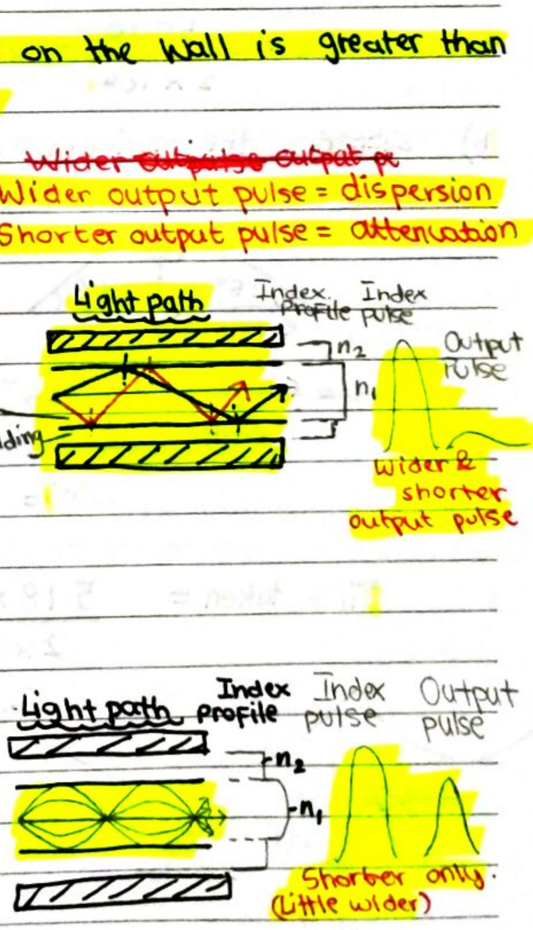
1) Step-index fibre (multi-mode step-index fibre)

- The refractive index of the core is constant, and so is of the cladding (but lower).
- Light can take different paths (Multi-mode) down the fibre as they have different wavelengths, and hence they undergo both material & waveguide dispersion. ~~(Later?)~~

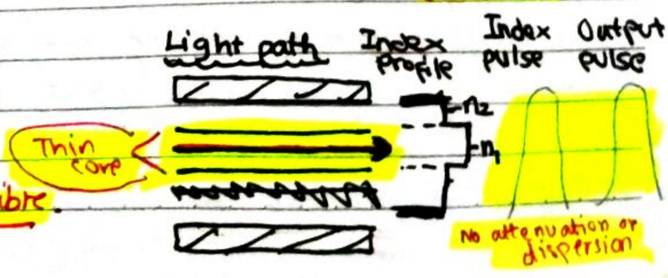


2) Graded-index fibre is an improvement.

It used a graded refractive index where n is highest at centre and gets lower towards the outer edge of the core. So, rays travel at different speeds depending on vertical distance ~~position~~ from centre. Cladding has constant n. (Spreading out is reduced of ~~the~~ pulse. Less dispersion. Most communications use this.)



3) The optimum fibre is Singlemode step-index fibre. Has a narrow core.



KIKY

Worked Example

C13 Length of an optical fibre is 5.0 km. Refractive index of core is 1.50 and critical angle of core-cladding boundary is 75°. Calculate the time taken for ray of light to travel down the length of the fibre:

a) Along a straight line parallel to the axis of fibre

~~$t = \frac{d}{v}$ $\frac{5000}{3\times10^8} = 1.7\times10^{-5}$ seconds~~

~~b) supposing maximum number of internal reflections in fibre.~~

Speed of light in core

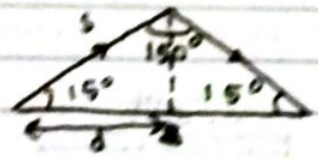
$n_1 v_1 = n_2 v_2$

$1 \times 3\times10^8 = 1.50\, v_2$

$v_2 = \frac{3\times10^8}{1.50} = 2\times10^8\ ms^{-1}$

$t = \frac{5000}{2\times10^8} = 25\ \mu s$

b) suppering the maximum number of internal reflections in the fibre.



$\theta_c = 75°$ So, θ_i is just larger than 75°

~~$s = \frac{d}{\sin\theta}$~~

$\sin 75° = \frac{d}{s}$

$s = \frac{d}{\sin 75°} = \frac{5}{\sin 75°} = 5.18\ km$

Time taken $= \frac{5.18\times1000}{2\times10^8} = 26\ \mu s$

~~C39~~

a) ~~$v_2 \sin\theta_1 = v_1 \sin\theta_2$~~

$n_1 V_1 = n_2 V_2$

$V_2 = \frac{3\times10^8}{1.6} = 2\times10^8$

$\frac{5000}{2\times10^8} = 25\ \mu s$

b)

$d = \frac{5}{\sin 75} = 5.18\ km$

$\frac{5180}{2\times10^8} = 26\ \mu s$

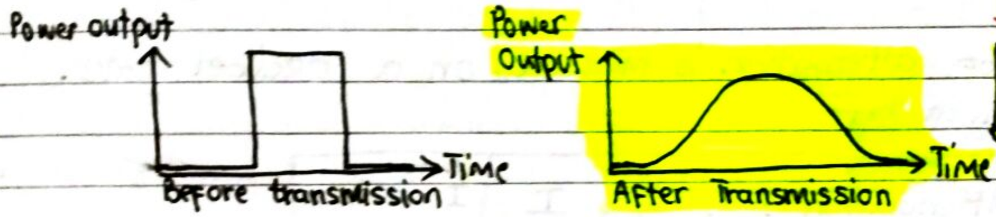
C3.2 Dispersion

We have two types of dispersion: (spreading of light-waves)

1) Material dispersion

- The refractive index of a medium depends on the wavelength of the light travelling through it. (As white light disperses in a prism).
- As light travels through optical fibre, different frequencies travel at slightly different speeds. Even if different wavelength rays follow the same path, they reach the end at different times.
- Effect of material dispersion
  A square wave spreads out in material dispersion.

Attenuation is smaller area.



- Lower height
- wider

~~But, dispersion is usually wider, and attenuation is lower.~~

2) Waveguide dispersion (Modal dispersion)

But light rays usually follow different paths.

- High-order mode: Rays that undergo many internal reflections follow this. (closer to critical angle)
  Low-order mode: Rays that undergo fewer reflections are in this mode.

- Consider rays with same wavelengths but different paths. The rays in low-order mode travel a shorter distance, and so reach the end faster than higher-order rays.

- Same effect on power-output as material dispersion.

→ Monomode (single-mode) step-index fibres ~~eliminate~~ reduce waveguide dispersion as it is so thin that it only allows one transmission path.

Exam tip: Graded-index fibres reduce waveguide dispersion as here, the speed of light away from the axis is also greater and so the longer path is covered at higher speed. The net effect is an almost same arrival time despite different paths.

Could be in the exam. IMPORTANT! How do different fibres overcome waveguide dispersion?

Date

C33 Attenuation

Definition :
- The loss in power of a signal ~~by~~ when travelling through a medium is called attenuation.

This attenuation is due to Scattering (dispersion) of light ~~and impurities~~ by glass molecules and impurities. (More later.) (Skip for now.)

The intensity at the end of the fibre is lesser as some energy is scattered or absorbed by the glass.

- The amount of attenuation depends on wavelength of light being transmitted. (Explained later)

The amount of attenuation is measured on a decibel scale. Attenuation is given by:

(dB) Attenuation $= 10 \log \frac{I}{I_0}$ (Power gain, Power loss = Attenuation)	Note: I is in Watts (Power)

I = ~~Output~~ Intensity of output power (Watts)
I_0 = Intensity of original input power (Watts)

I is in Watts. So, kinda like power, not intensity.

Negative attenuation means Output power < Input power.
Positive attenuation means amplification. Output power > Input power

Attenuation per unit length is in $dBkm^{-1}$.

Attenuation per unit length = $\frac{\text{Attenuation}}{\text{Length of Fibre}}$ (aka specific Attenuation)	L is in kilometers (KM)

Worked Examples

C.14 An amplifier amplifies an incoming signal of power 0.34 mW to 2.2mW. Calculate power gain of amplifier in decibels.

Sol. Power gain refers to Attenuation

$$A = 10 \log\left(\frac{I}{I_0}\right) = 10 \log\left(\frac{2.2 \times 10^{-3}}{0.34 \times 10^{-3}}\right)$$

$$A = 8.1 \text{ dB}$$

Date

C.15 A signal of power 12 mW is input into a cable of attenuation per unit length 4.0 $dB\,km^{-1}$. Calculate signal after it has travelled 6.0 km in cable.

$$A = -4.0 \times 6.0 = -24\,dB$$

$$A = 10\log\left(\frac{I}{I_0}\right) \rightarrow -24 = 10\log\left(\frac{I}{12\times10^{-3}}\right)$$

~~$24 = 10\log\left(\frac{12\times10^{-3}}{I_0}\right)$~~

$$-2.4 = \log\left(\frac{I}{12\times10^{-3}}\right)$$

~~$24 = \log\left(\frac{12\times10^{-3}}{I_0}\right)$~~

$$(10^{-2.4})(12\times10^{-3}) = I$$

$$I = 4.8\times10^{-5}\,W$$

$$= 0.048\,mW$$

- We assume A to be negative if question doesn't say that signal is amplified. A is negative unless signal is amplified. So, if it says 30 dB in question, use −30 dB. Same for M of compound microscope.
- Also, it is good to remember that if output power doubles, attenuation is +3, and if output power halves, attenuation ~~drops to one-third~~ is −3.

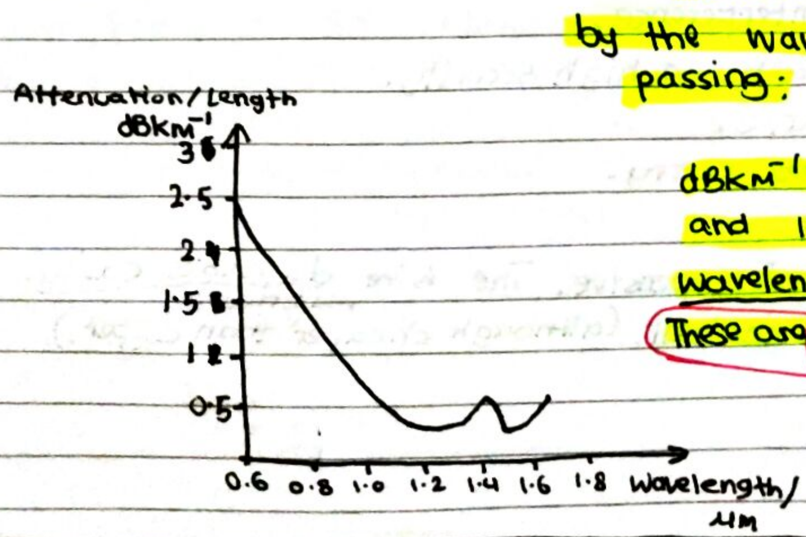
$$A_1 = 10\log 2 = 3.01 \approx 3\,dB \qquad A_2 = 10\log\left(\tfrac{1}{2}\right) = -3.01 \approx -3\,dB$$

Why attenuation occurs? – Main: Scattering of light off impurities in the core.

1) Impurities in glass. ~~(State other 2 first)~~
2) Scattering of light in the glass.
3) Absorption of light by the glass.

1 is connected to 2. So, it is just one reason basically.

These 2 factors are affected by the wavelength of light that is passing:

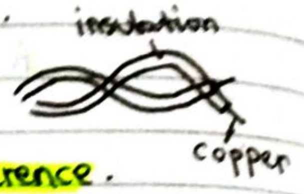


$dB\,km^{-1}$ is minimum at 1310 nm and 1550 nm. These are desirable wavelengths for transmission. These are infrared wavelengths.

Infrared wavelengths suffer low attenuation.

C 3.4 Advantages of optical fibres

1) Twisted pairs of copper wires were first used.



Advantages : 1) Simple and cheap.

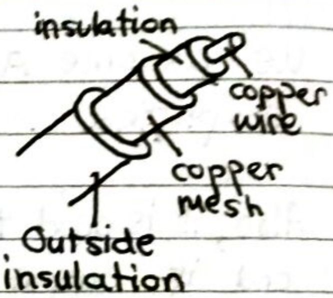
Disadvantages : 1) Susceptible to noise and interference.

2) Unable to transfer info. at high rate.

Use : Intercoms (simple systems)

2) Coaxial cables were then introduced.

A central wire is surrounded by the second wire in the form of an outer cylinderical copper tube or mesh. An ~~i~~ insulator separates the two wires.



Advantages 1) Straightforward & simple.

2) Lesser noise & interference.

Disadvantages 1) Expensive (relatively).

Use : TV.

3) Optical fibres

Laser light can be sent to transfer signals.

Use : Long-distance telecommunication, high volume data (video)

Advantages

1) Lower attenuation.
2) Higher transmission capacity.
3) No electromagnetic interference.
4) Negligible cross talk. So, high security.
5) No sound disturbance.
6) Low weight. Easy to carry.

Disadvantages

1) Repairing is complex & expensive. The wire damages easily.
2) ~~Expen~~ Expensive to install. (although cheaper materials than copper.)

Date

Test yourself

42) Calculate speed of light if refractive index is 1·45 of core.

$$n_1 v_1 = n_2 v_2$$

$$1 \times 3 \times 10^8 = 1{\cdot}45 \times v_2$$

$$v_2 = \frac{3 \times 10^8}{1{\cdot}45} = 2{\cdot}07 \times 10^8 \ ms^{-1}$$

43) a) Total Internal Reflection

- The phenomena when light travelling from a optically denser medium to an optically less dense medium gets completely reflected and no light is refracted through.

b) Critical angle

The angle of incidence at which the angle of refraction is perpendicular to the normal when light travels from an optically denser medium to an optically less dense medium.

c) Why ~~does~~ TIR only occurs from high-to-low refractive index medium and not vice-versa?

Critical angle is found by $n_1 \sin\theta_1 = n_2 \sin 90°$, $\sin c = \frac{n_2}{n_1}$.

If $n_2 < n_1$, $\sin c < 1$, which is impossible. Hence, total internal reflection is a one-way phenomenon.

44) Core has n = 1·50. Cladding has n = 1·46. Calculate critical angle in the optic fibre at the core-cladding boundary.

$$n_1 \sin c = n_2 \sin 90°$$

$$\sin c = \frac{n_2}{n_1} = \frac{1{\cdot}46}{1{\cdot}50}$$

$$c = 76{\cdot}7°$$

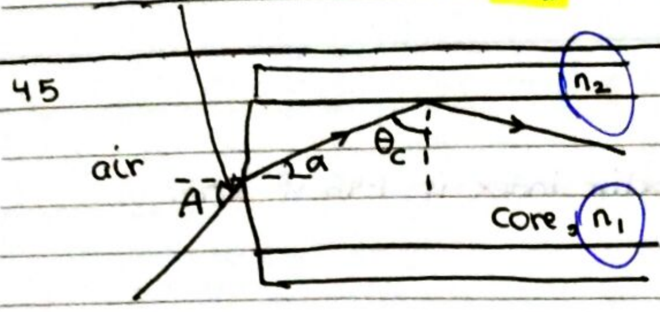
$\sin A = \sqrt{n_1^2 - n_2^2}$ (core, cladding)

n_1 = For core
n_2 = For cladding
$\sin A = \sqrt{n_1^2 - n_2^2}$

ACCEPTANCE ANGLE! $A = \arcsin\sqrt{n_1^2 - n_2^2}$

Date

45



a) Show that $\cos\theta_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$.

Solution

$n_1 \sin\theta_c = n_2$

$\sin\theta_c = \frac{n_2}{n_1}$

$\sin^2\theta_c + \cos^2\theta_c = 1$

$\cos^2\theta_c = 1 - \frac{n_2^2}{n_1^2}$

$\cos\theta_c = \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$

$\cos\theta_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$

b) Show that the max. angle; A from air that will lead to TIR is given by $A = \arcsin\sqrt{n_1^2 - n_2^2}$.

$n_{air} \sin A = n_1 \sin\alpha$

$1 \sin A = n_1 \sin\alpha$

$\sin\alpha = \sin(90° - \theta_c)$

$= \cos\theta_c$

$\sin\alpha = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$

$\sin A = \frac{~~n_1~~\sqrt{n_1^2 - n_2^2}}{~~n_1~~}$

~~A = ∫~~

$A = \arcsin\sqrt{n_1^2 - n_2^2}$

c) Calculate acceptance angle A of an optical fibre with n_1 1.50 and n_2 1.40.

$A = \arcsin\sqrt{1.5^2 - 1.4^2}$

$A = 32.6°$

$\theta_c = \arcsin\left[\frac{1.40}{1.50}\right] = 68.96°$

$90 - 68.96 = 21.0°$

$\sin A = 1.50 \sin 21$

$A = 32.5°$

47) Refractive index of cladding of an optical fibre is 1.42. Determine refractive index of core such that any ray entering the fibre gets totally internally reflected.

~~$n_1 \sin\theta_1 = 1.42 \times \sin 90$~~

Even when ~~$\sin\theta_1 = \frac{1.42}{n_1}$~~

$A = 90°$, it should TIR

~~By any ray, it means any angle of incidence from θ.~~

~~$\sin\theta = \frac{1.42}{n_1}$~~

~~n_1 must be ∞. Exceptionally high refractive index.~~

$A = \arcsin\sqrt{n_1^2 - n_2^2}$

$90° = \arcsin\sqrt{n_1^2 - n_2^2}$

$1 = \sqrt{n_1^2 - 1.42^2}$

$n_1 = 1.74$

8) State one crucial property of the glass in the core of an optical fibre.

It must be free of impurities.

Date

49 a) What is meant by dispersion in context of optical fibres.

Solution

The phenomenon where speed of a wave depends on wavelength. Different wavelengths take different times to travel the same distance.

b) Distinguish between material & waveguide dispersion.

Solution

Material dispersion occurs when different wavelengths reach the end of an optic fibre at different times as the core medium has a unique refractive index for each wavelength. Waveguide dispersion occurs when light rays of without the same wavelength travel along different paths, high-order mode or low-order mode, and reach the end of the optic fibre at different times.

50) Optic fibre has length 8.00 km. Core has refractive index 1.52 and core-cladding critical angle is 82°.

a) Calculate speed of light in core.

~~$n_1 v_1 = n_2 v_2$~~

$n_{air} \times 3\times10^8 = 1.52 \times v_2$

$\frac{3\times10^8}{1.52} = v_2$

~~$n_1 \sin\theta_1 = n_2 \sin 90°$~~

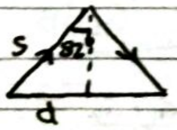
~~$1.52 \times \sin 82 = n_2$~~

~~$n_2 = 1.51$~~

$v_2 = 1.97 \times 10^8\ ms^{-1}$

b) Calculate min and max times for ray of light to travel down the length of the fibre.

$T_{min} = \frac{8000}{1.97\times10^8} = 4.05\times10^{-5}$ seconds



$\sin 82° = \frac{d}{s}$

$s = \frac{d}{\sin 82} = \frac{8000}{\sin 82} = 8079\ m \approx 8080\ m$

$\frac{8080\ m}{1.97\times10^8} =$ ~~4.10x10^-5 sec~~ $= 4.09\times10^{-5}$ sec

(KIKY)

Good formula at the end of page!

Date

52 a) Distinguish between monomode and multimode optical fibres.

Solution

A monomode optic fibres has a very thin core so that all rays entering it follow the same path. Multimode fibres allow different paths of different length.

b) Discuss effect of reducing fibre core diameter on bandwidth that can be transmitted by fibre.

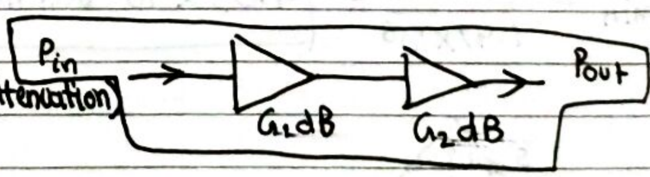
Solution (discuss)

There is a large increase in bandwidth as there are lower effects of dispersion and hence transmission frequency increases. Waveguide dispersion is eliminated in a single-mode fibre with small diameter and material dispersion can be reduced by using monochromatic light or laser. Hence, bandwidth is increased as core diameter is ~~used~~ reduced.

53) Three advantages of optic fibres in communication.

1) Low chances of cross-talk and so higher security.
2) Low attenuation. ~~as waveguide dispersion~~
3) Large bandwidth and so video and other large data is transferred at great speed.
4) No noises.
5) Thin and light.

55) Two amplifiers of gain G_1 and G_2 (in dB) amplify a signal. Calculate overall gain produced by 2 amplifiers.



Solution (overall gain = overall attenuation)

$G_1 = 10 \log\left(\frac{I'}{I_0}\right)$

$\frac{G_1}{10} = \log\left(\frac{I'}{I_0}\right)$

$I_0 \times 10^{\frac{G_1}{10}} = I'$

$G_2 = 10 \log\left(\frac{I''}{I_1}\right)$

$10^{\frac{G_2}{10}} = \frac{I''}{I_0 \times 10^{\frac{G_1}{10}}}$

$I_0 \times 10^{\frac{G_1+G_2}{10}} = I''$

$\frac{G_1+G_2}{10} = \log\left(\frac{I''}{I_0}\right)$

$G_1 + G_2 = 10 \log\left(\frac{I''}{I_0}\right)$

So, overall gain $= G_1 + G_2$. Can use in ~~Q54~~ Q60.

KIKY

Date

57) Signal of power 8.40 mW is attenuated to 5.10 mW over 25 km cable. Calculate attenuation per unit length of cable.

<u>Solution</u>

$$A = 10 \log\left(\frac{5.10 \times 10^{-3}}{8.40 \times 10^{-3}}\right)$$

$$A = -2.17 \text{ dB}$$

$$\frac{A}{25} = \frac{-2.17}{25} = -0.0867 \text{ dB km}^{-1} \approx 0.087 \text{ dB km}^{-1}.$$

58) A coaxial cable has specific attenuation 12 dB km⁻¹. Signal must be amplified when power falls to 70% of input power. At what distance signal must be amplified.

<u>Solution</u>

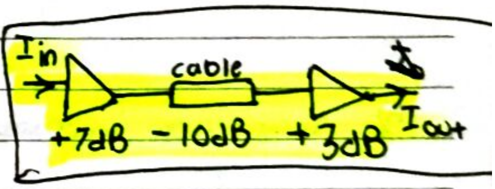
Take A as negative.

$$-12 \times L = 10 \log\left(\frac{0.7 I_0}{I_0}\right)$$

$$-1.2 L = \log 0.7$$

$$L = \frac{\log 0.7}{1.2} = 0.13 \text{ km}$$

60) A signal is input into amplifier of gain +7.0 dB. Then it suffers power loss 10 dB by travelling in a cable, and then amplified by gain +3.0 dB. Calculate ratio of ~~input and ou~~ output power to input power.



$$7.0 - 10 + 3 = 0 \text{ dB} = \text{Attenuation overall.}$$

$$0 = 10 \log\left(\frac{I}{I_0}\right)$$

You can get overall Attenuation by adding them together. + for amplifier, − for attenuation

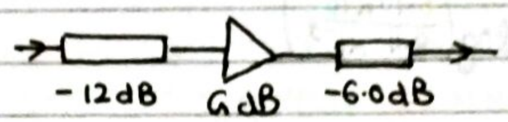
$$1 = \frac{I}{I_0}$$

Ratio = 1

(If I was 2 times larger than I_0, A = 3. Good to remember.)(Next question.)

Date

61) Output power is twice the input power. Calculate required gain G of amplifier. So, total A is positive.



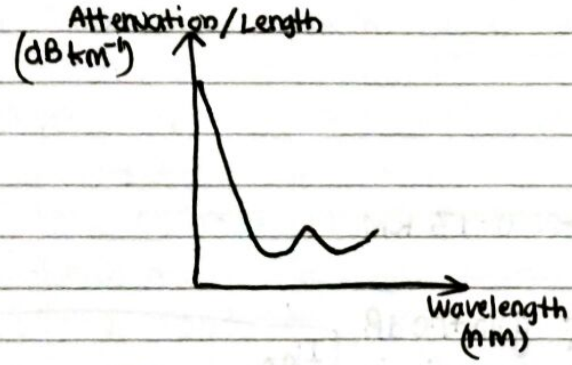
IF $2I = I_0$, $A = 3$ because $3 = 10\log 2$.

$A = -12 + G - 6 = -18 + G$

$3 = -18 + G$

$G = 21$ dB

62) a) Sketch graph (no numbers) to illustrate variation with wavelength of specific attenuation in an optical fibre.



b) Why are infrared wavelengths optimal for optic fibre transmission.

The wavelengths with lowest specific attenuation are infrared wavelengths. Hence, these are optimum.

Time for HL part. Mostly descriptive but has some formulas! :)

At least there is only one HL unit. So, that is good. Other options have more.

KIKY

C.4 Medical Imaging (HL)

Date

L.O.

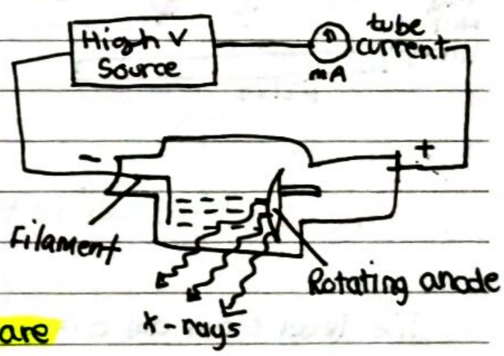
1) X-rays - attenuation coefficient, half-value thickness, linear/mass absorption, improvements in contrast and sharpness
2) Solve ultrasound acoustic impedance, speed of ultrasound through tissue and air and relative intensity levels.
3) Features of ultrasound techniques, choice of frequency and A & B scans
4) Gradient fields in NMR
5) Advantages & disadvantages of NMR and ultrasound. Risk factors.

C4.1 X-ray Imaging

X-rays are electromagnetic radiation with a wavelength of 10^{-10} m.

X-rays are produced in X-ray tubes, where electrons are accelerated to high energies by high potential differences. They then collide with a metal target. As a result of the deceleration in the electrons and transitions between energy levels in the target atoms, X-rays are emitted.

First radiation invented in medical imaging.



Attenuation - the energy loss of X-rays when travelling through a medium.

- Photoelectric effect is in use. X-ray photons are absorbed by electrons in the medium and electrons receive energy. The effect depends on atomic number of the elements present in bone (Z=14) and in soft tissue (Z=7). Bone absorbs X-rays more strongly than soft tissue. Hence, X-ray image will show a contrast between bone and soft tissue.

IMPORTANT PARAGRAPHS ABOUT CONTRAST AND SHARPNESS !!!

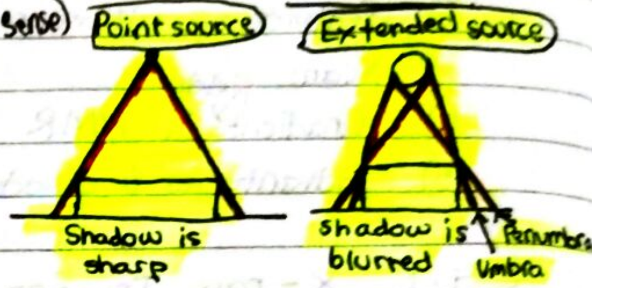
If there is no difference between Z numbers, example- in digestive tract, the image is improved by using a contrast medium. (It is simple. DW) The patient swallows this, usually barium sulfate, and barium absorbs X-rays more strongly than surrounding tissue. The image created by X-rays on the film is thus a shadow of the high-Z material against surrounding low-Z tissue.
(Film is below patient btw.)

- To increase sharpness of the shadow, the film should be as close as possible to patient or source should be far from patient.

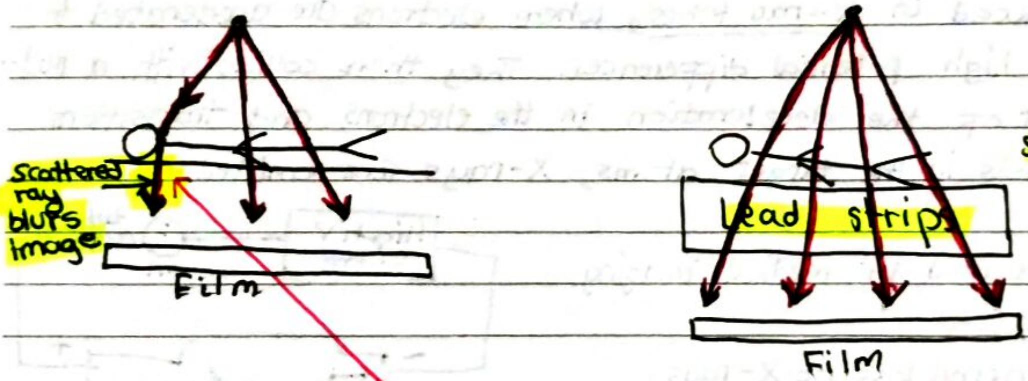
Here, intensity of X-rays can diminish, so they require longer exposure time.

Basically, for best sharpness, the source should be point-like.

1) Put film close to patient. (Makes sense)
2) Put source far from patient.



- Image is also improved if scattered rays are prevented from reaching the film. Use lead strips between the patient and film as lead readily absorbs X-rays.



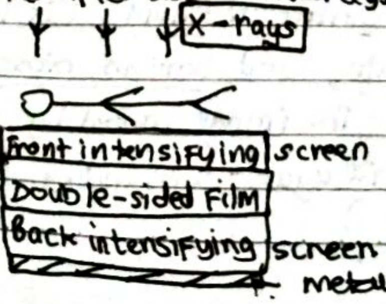
Using a grid of lead strips blocks scattered rays, improving the image.

The scattered ray is the one that changed direction.

The lead strips are 0.5mm apart, and are closely oriented in direction of incoming x-rays, so that scattered rays are absorbed.

- Low energy X-rays are absorbed by patient's skin and therefore of no use. They are filtered out from incoming beam.

- The exposure time for an x-ray image is long as photographic film is more sensitive to visible light. But, we intensify the screen by using a double-sided photographic film. The crystals in intensifier absorb x-rays and re-emit as visible light for the film.

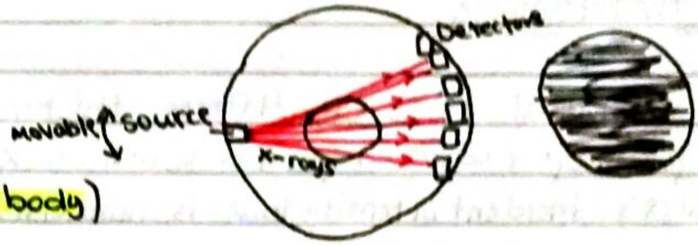


KIKY

Date

C4·2 Computed Tomography

- Uses x-rays.



Advantage
1. More accurate diagnosis
2. Takes little time (6 seconds for body)

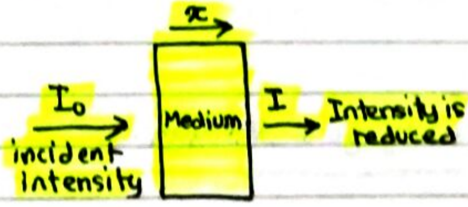
What happens? (look at image)

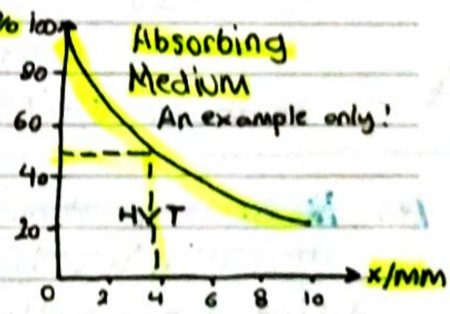
A movable X-ray source emits a beam at right angles to the patient, which detector detects on the other side. Using many detectors reduce exposure level & makes scan faster.

The source and detectors are rotated & moved around body, and data is combined into a 3-D computer image, viewable as 2-D 'slices' at any position.

C4·3 Attenuation

- Image ~~CT sca~~ x-rays of intensity I_0 incident on a medium normally. After travelling distance x through medium, intensity of x-rays decreases to I.





- Similar to half-life & decay.

$I = I_0 e^{-\mu x}$	μ = Attenuation coefficient, unit = distance^{-1}	Formula for Attenuation

- μ is attenuation coefficient. This depends on ① density ρ of material (medium). ② Atomic no. Z of material (medium). ③ Energy of the x-ray photons.
- **Half-Value Thickness**: the penetration distance at which intensity is halved. (HVT) ($x_{\frac{1}{2}}$)

$$\frac{I_0}{2} = I_0 e^{-\mu x_{\frac{1}{2}}}$$

$$\frac{1}{2} = e^{-\mu x_{\frac{1}{2}}}$$

$$\ln\frac{1}{2} = -\mu x_{\frac{1}{2}}$$

$$(-\ln\frac{1}{2} = \ln 2)$$

$$\frac{\ln 2}{\mu} = x_{\frac{1}{2}}$$

(Similar to half-life $t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$, λ = decay constant)

$x_{\frac{1}{2}} = \frac{\ln 2}{\mu}$	Half-Value thickness	Not in D.B.

Date

The higher the energy in x-rays, the higher the half-value thickness.

Worked Example

C.17 A metal sheet of 4.0mm thickness and HVT 3.0mm is placed in path of radiation from a source of x-rays. Calculate fraction of source's incident intensity that is transmitted through the sheet.

$$I = I_0 e^{-\mu x}$$

$$\mu = ?$$

$$x_{\frac{1}{2}} = \frac{\ln 2}{\mu}$$

$$3.0\,mm = \frac{\ln 2}{\mu}$$

$$\mu = \frac{\ln 2}{3.0} = 0.231\,mm^{-1}$$

$$I = I_0 e^{-0.231 \times 4}$$

$$\frac{I}{I_0} = 0.397$$

$$I = 0.397 I_0$$

or about 39.7% of incident intensity goes through.

- For a given atomic number Z and energy of x-ray photons E, the attenuation coefficient is proportional to density. Through this, we define a new coefficient - mass absorption coefficient :

Guide: Factors affect μ. Previous page.

$$\mu_M = \frac{\mu}{\rho}$$

Mass absorption coef.
~~Not in Booklet~~
ρ = density of medium
μ = attenuation coefficient.

Hence, we can compare between attenuations in materials with different densities.

Date

C 4·4 Ultrasound

~~c = 1540 ms⁻¹ here. (Speed of sound in tissue)~~

Ultrasound means sound inaudible for human ear, with frequency > 20000 Hz.
About 1–10 MHz is frequency of ultrasound.

- **Advantage** – No radiation damage in body or other side-effects. No ionization. Used with pregnant women.

  Disadvantage – Images are not as detailed as X-rays.

- What happens?

  The ultrasound is emitted towards patient's body in short pulses, lasting 1 μs, and their reflections off different organs are detected. A 1 μs pulse of 1 MHz sound contains a single wavelength, while a 1 μs pulse of 10 MHz sound contains 10 wavelengths.

For example, speed of sound in soft tissue is $1540\,ms^{-1}$. If $f = 1$ MHz for ultrasound,
$\lambda = \frac{1540}{1000000} = 1.54$ mm. If $f = 10$ MHz, $\lambda = 0.154$ mm.

In general, (not in ultrasound)

- Diffraction places a limit on the size d that can be resolved using a wavelength λ. The ~~wavelength~~ constraint is:

$\lambda < D$	Constraint due to diffraction

D = separation distance of 2 objects

So, if resolution of few mm is needed, the wavelength must be lesser than those few mm.

> For the ultrasound frequencies used in medicine, it is the pulse duration, not the diffraction, that sets limits on resolution.

- Different organs require different frequencies.

$f = 200\frac{c}{d}$	Frequency in ultrasound depending on organ
c = speed of sound in tissue = $1540\,ms^{-1}$	
d = depth of the organ below body surface	

You should get an answer in single order of MH. Like 1-10 MH

Q) The stomach is about 10 cm from body's surface. Suggest what frequency should be used for an ultrasound scan of the stomach?

Solution.

$f = \frac{200 \times 1540}{0.1}$

$f = 3 \times 10^6$ Hz = 3 MHz

Set
CHOICE OF FREQUENCY (Important)
High f means high resolution, but high attenuation. Hence, there is a trade-off. Optimum f is when organ is 200 wavelengths far.

3 Important formulas on this page! → acoustic impedence, relative intensity levels

- Transducer is the source of ultrasound. It converts electrical energy into sound energy, using the phenomenon piezoelectricity. Also known as probe. it emits pulses of ultrasound.
- Transducer or probe also acts as the receiver of ultrasound.
- The sound energy produced is directed into the patient's body. Usually, when a wave encounters an interface between two mediums, part of the wave is reflected and part of it is transmitted.

Degree of transmission depends on acoustic impedence of the two media. Acoustic impedence (Z) is:

- $Z = \rho c$ — Acoustic Impedence

Units of $Z = kg\,m^{-2}s^{-1}$

ρ = density of the medium

c = speed of sound in that medium (Not light speed)

IMPORTANT:

$\frac{I_t}{I_0} = \frac{4Z_1Z_2}{(Z_1+Z_2)^2}$	Fraction of intensity transmitted
$\frac{I_r}{I_0} = \frac{(Z_1-Z_2)^2}{(Z_1+Z_2)^2}$	Fraction of intensity reflected

- If the mediums have very different acoustic impedences, most of the energy is reflected. Hence, low difference in Z is needed for more energy to be transmitted. This is called impedence matching.
- Why is a gel-like substance applied during ultrasound?

The impedence between air and soft tissue is huge, so almost all energy would be reflected with an air-tissue interface. So, the space between the body and transducer is filled with a gel-like substance whose acoustic impedence is close to tissue.

Worked Example (Good Example to understand this concept)

C.19. Density of air is $1.2\,kg\,m^{-3}$ and speed of sound is $340\,ms^{-1}$. Density of soft tissue is $1200\,kg\,m^{-3}$ and speed of sound is $1500\,ms^{-1}$. a) Calculate impedences. b) Calculate fraction of intensity of ultrasound that would be transmitted from air to soft tissue. c) Comment on your answer.

a) $Z_a = 1.2 \times 340 = 408\,kg\,m^{-2}s^{-1}$. $Z_T = 1200 \times 1500 = 1800000\,kg\,m^{-2}s^{-1}$.

b) $\frac{I_T}{I_0} = \frac{4Z_1Z_2}{(Z_1+Z_2)^2} = \frac{4(408)(1800000)}{(1800408)^2} = 0.0009 = 9 \times 10^{-4}$

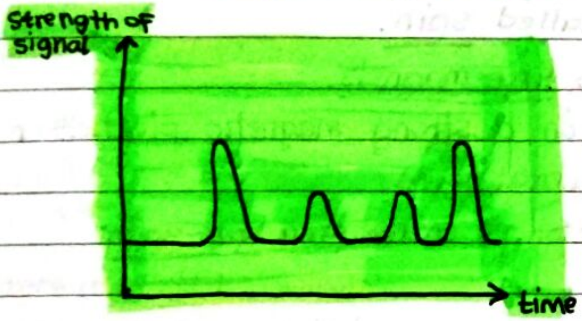
c) Negligible amount of ultrasound is transmitted and most is reflected. This shows the need of a suitable gel between transducer and skin.

Date

A- and B- scans

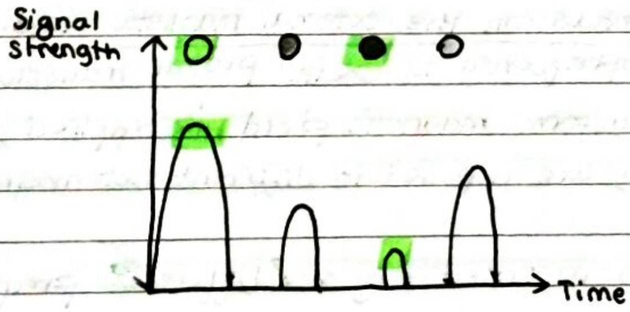
There are two ways to present the data gathered by probe < the A-scan / the B-scan

A - scan	B - scan
1) Amplitude - modulated scan	1) Brightness - modulated scan
2) Graph of signal strength versus time.	2) ~~Graph of~~ Dots of light on a screen which depends on signal strength. (with reference to left graph) see below:



• 2-D image.

• Real - time ultrasound live video.



↑ higher signal strength, ↑ higher brightness of dots.

~~Ultrasound can produce a 2-D image of the surface of the organ~~

Finally, last part of imaging is here. Magnetic Resonance Imaging (MRI)

C4.5 Magnetic Resonance Imaging

Date

- Magnetic resonance imaging (MRI) is based on a phenomenon called nuclear magnetic resonance (NMR).

- Similar to Computer Tomography (CT), but it doesn't use dangerous radiation. However, very expensive. (Used to detect brain-tumours.)

(Remember the process!)

In involves the use of a non-uniform magnetic field in conjunction with a large strong uniform field. The process is as follows:

1) ~~Nuclei~~ Electrons & protons of atoms have a property called spin.
2) The spin means these ~~Nuclei~~ protons acts as tiny magnets. But, random direction so, net B field is 0.
3) These ~~Nuclei~~ protons will ~~tend~~ to line up in a strong magnetic field either parallel or anti-parallel to the external field. so, there is a B-field now in body
4) A radiofrequency signal forces proton to change from spin-up to spin-down state.
5) The proton de-excites returns to spin-up state by emitting a photon of same frequency as the radio frequency signal which is detected by detector. (Larmor Frequency)
6) This frequency depends on the external magnetic field strength. Different field requires different radiofrequency to excite proton transitions.
7) A secondary, non-uniform magnetic field is applied so that different parts of body are exposed to different net magnetic fields. (Makes sense)
8) Each part of body is revealed by a different frequency of emitted photons.
9) The rate of these transitions also gives information about tissue type.

- Larmor frequency - the rate of precession of the magnetic moment of the proton around an external magnetic field.

  Precession is the oscillation of protons when they are lining up due to the external magnetic field.

  Precession - Protons spin around ~~primary~~ axis of magnetic field when they are lining up.

- Spin-up state has lower energy the ~~n~~ spin-down state.

Date

In terms of patient and machine: (good to understand)

1) The patient lies in an enclosure surrounded by powerful magnet that creates a uniform magnetic field.
2) An additional non-uniform magnetic gradient field is superimposed on this, so that total field varies across the patient.
3) Now, a magnetic field which is different in different body parts means different frequencies of resonant photon absorption and re-emission.
4) Variation of frequency produces a three-dimensional computer image.

1. Explain the use of gradient field?

- They determine the point from which photons are emitted.

2. Proton spin relaxation time - the rate at which excited protons return to their lower states.

- Explain the origin of the relaxation of proton spin and consequent emission of signal in NMR.

Nuclei in the excited state must be able to "relax" and return to their ground state.
The timescale for relaxation will dictate how the NMR experimented is executed and consequently, how successful it is.

Comparison between Ultrasound and NMR Imaging

Ultrasound	NMR
+ Cheaper, and lighter equipment so equipment can be taken to patient for comfort.	- More expensive. Equipment can't be carried as very bulky.
+ No radiation dangers.	+ No radiation dangers.
- Low quality images. Low resolution.	+ Better resolution images.
+ No claustrophobia.	- Difficult for claustrophobic patients.
- Some organs are not accessible.	+ Can distinguish between different tissues.
+ Patient can move.	- Patient must be still.
+ No danger ~~wit~~ for patients with metal-implants.	- Dangerous for patients with metal-implants as magnetic field can attract metal.

N.O.S. Radiation danger is present and patient can get cancer. Doctor must evaluate the dangers and sickness before deciding which imaging technique should be optimal.

(KIKY)