10. FIELDS

10.1 Describing Fields (Extension of 5.1 and 6.2)

Date

- Essential idea: Electric charges, and masses influence the space around them and this influence can be represented through fields.

- Understandings:
1) Gravitational fields
2) Electrostatic fields
3) Electric potential and gravitational potential
4) Field lines
5) Equipotential surfaces - Remember this?

Application and skills

- Representing masses and charges, field lines and patterns using symbolism
- Mapping fields using potential.
- Describe connection between equipotential surfaces and field lines.
-

Guidance

1) No work is done in moving a charge or mass on an equipotential surface.
2) Only three types of electrostatic fields: Around point charges, Between 2 point charges, between charged parallel plates
3) Only two types of gravitational fields: radial fields around point or spherical masses, the (assumed) uniform field close to surface of massive celestial bodies

Data booklet (Memorize other formulas)

- $W = q \Delta V_e$
- $W = m \Delta V_g$

1). Conservative Forces (Important and simple)

$\boxed{W = Fd\cos\theta}$ θ is angle between F & d.

- The work done by gravity (g) is independent of the path over which it is done. It simply depends on vertical displacement.

- Conservative force - Any force which does work independent of a path. E.g. gravitational force.

Friction is not conservative. Depends on path.

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Only conservative forces have associated potential energy functions.

Makes sense! Since gravity's work must be balanced ~~so a total 0 net work~~ by another work.

So, if work is done in a conservative force field, there is an associated potential energy function.

$\Delta E_p = -W = -Fd\cos\theta$ where F is a conservative force	Potential energy function

How?

Conservation of E: $\Delta E_p + \Delta E_K = 0$

work-kinetic theorem: $W = \Delta E_K$

$$\Delta E_p + W = 0$$
$$\Delta E_p = -W = -\Delta E_K$$

Hence, if $W = Fd\cos\theta$, $\Delta E_p = -Fd\cos\theta$.

So, the interesting part:

Work is a scalar but can be negative if F & d are in opposite directions.

Good Example!

If a box is lifted upwards, $F = mg$, and $d =$ upwards. But g is downwards and d is up, hence $W = Fd\cos 180 = -Fd = -mgd$.

Now, if = $\Delta E_p = -W$, $\Delta E_p = -(-W) = +(mgd) = mgd$. ΔE_p is positive.

WHEN AN OBJECT is LIFTED vertically UPWARDS, $\Delta E_p = +$ (positive) and $W = -$ (negative).

Both electrostatic force and gravitational force are conservative. Hence, they both have potential energy functions (Topic 10.2)

$F_e = \frac{kq_1q_2}{r^2}$ $k = 8.99 \times 10^9 NM^{2}C^{-2}$	Coulomb Law

But we take it as positive.

$F_g = \cancel{-}\frac{GM_1M_2}{r^2}$ $G = 6.67 \times 10^{-11} NM^2kg^{-2}$	Universal law of gravitation

F_e is negative if q_1 and q_2 are opposite (Attractive)

Notice the negative sign?

Remember it! Attractive force always negative. Take it as positive only for force.

$F = -kx$	Another conservative force.

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ATTRACTIVE Force is Negative

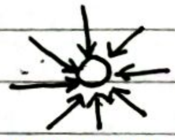
When two + or − charges, positive. When different signs, negative.

≥ F_g is always negative (−).

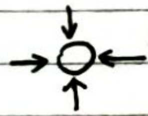
($F = -kx$) (For remembrance)

2). <u>Force Fields</u>

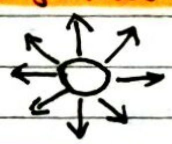
- The larger the mass of an object, the stronger the gravitational field around it.



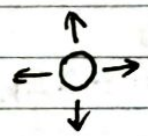
Large mass (− charge)



Small mass (− charge)



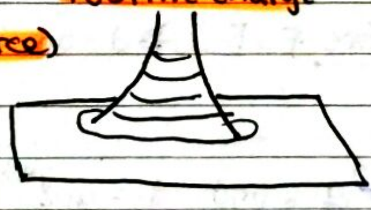
Large mass (+ charge)



≥ Small mass (+ charge)

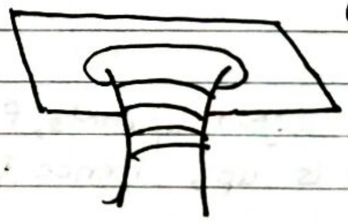
- Positive charge

(Repel force)

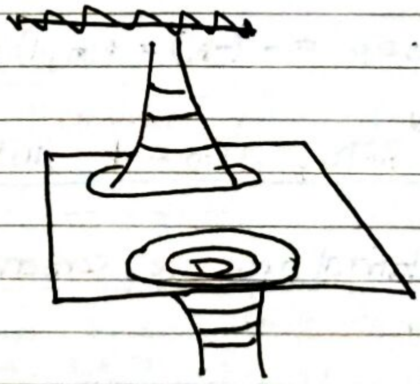


Negative charge, a mass

(Attractive force)



Fall in!



An electric dipole.

A gravitational dipole does not exist.

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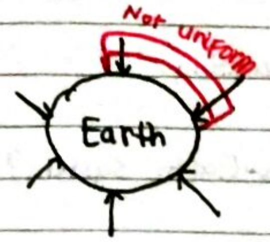
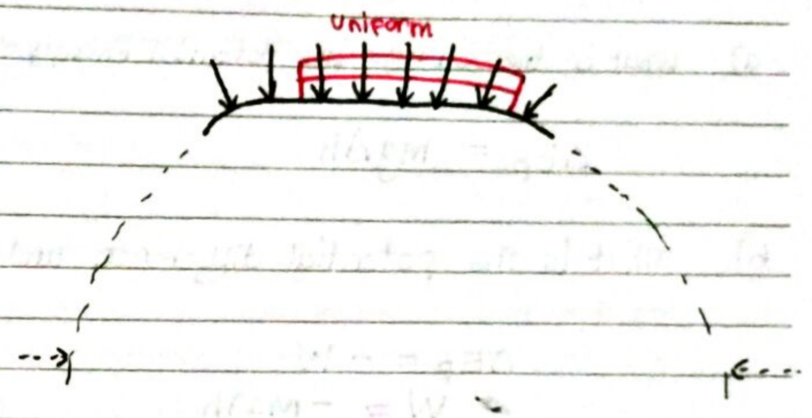
- When we are closer to Earth's surface, the gravitational field is more uniform.

a) Far



b) On Earth



- Between two parallel plates, electric field strength is uniform.

3). Potential difference - the electric force and gravitational force

①. Potential difference ΔV_e - The amount of work (W) done per unit charge (q) in moving a point charge from A to B.

$\Delta V_e = \frac{W}{q}$	Electrostatic potential difference JC^{-1} or V

Formula 1

Example

A $+15.0\ \mu C$ charge q is moved from Point A, with voltage (potential) 25.0V to point B, with voltage (potential) of 18.0V.

a) ~~Potential difference~~ What is the work done in moving charge q from A to B.

$\Delta V_e = 18 - 25 = -7V$

$W = \Delta V_e \times q = -7 \times 15 \times 10^{-6} = -1.05 \times 10^{-4} J$

②. Potential difference ΔV_g - The amount of work (W) done per unit Mass (m) in moving a point mass from A to B.
(cant put V as unit. Only Jkg^{-1}!)

$\Delta V_g = \frac{W}{m}$	Gravitational potential difference Jkg^{-1}

Formula 2

Example

A mass of M = 500kg is moved from point A, with gravitational potential of $75.0 Jkg^{-1}$ to point B, having a gravitation potential of $25.0 Jkg^{-1}$

a) Work done to move it from point A to point B?

$\Delta V_g = 25.0 - 75.0 = -50.0\ Jkg^{-1}$

$W = \Delta V_g \times m = -50.0 \times 500 = -25000 J = 25 \times 10^{3} J$

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Practice (Good one for theoretical understanding)

A mass m moves upward a distance Δh without accelerating.

a) what is the change in potential energy of the mass-Earth system?

$$\Delta E_p = mg\Delta h$$

b) What is the potential difference undergone by the mass?

$$\Delta E_p = -W$$

$$W = -mg\Delta h$$

$$\Delta V_g = \frac{W}{m} = \frac{-mg\Delta h}{m}$$

$$\Delta V_g = -g\Delta h$$

c) what is the gravitational field strength g in terms of V_g and Δh?

$$\Delta V_g = -g\Delta h$$

$$g = -\frac{\Delta V_g}{\Delta h}$$

d) What is the potential difference experienced by mass moving from $h = 1.25m$ to $h = 3.75m$? Use $g = 9.81\ ms^{-2}$.

$$\Delta V_g = \frac{W}{m}$$

$$W = -E_p = -m \times 9.81 \times 2.5 = -24.525m\ J$$

$$\Delta V_g = \frac{-24.525m}{m} = -24.525\ J\,kg^{-1} \approx 24.5\ J\,kg^{-1}$$

We get a formula in c) that

$g = \frac{\Delta V_g}{\Delta h}$	Field strength = $\frac{-\text{potential difference}}{\text{Position change}}$

Date

4). Equipotential surfaces – the gravitational field

Self-explanatory name

- Equipotential surface – a surface with a constant gravitational potential. No potential difference along the surface.

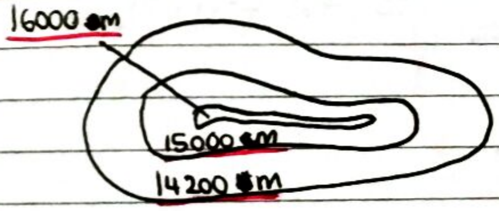
~~Wtf~~ See, $\Delta V_g = -g\Delta h$ (In previous question)

If h is constant, so is V_g.

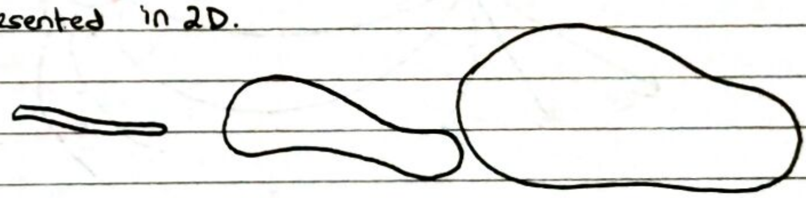
Hence, at every height, there is an equipotential surface. If a mass only moves horizontally, there is no potential difference and thus no work done by gravitational force. $\Delta V_g = 0$, $W = 0$

~~For nearby stuff~~ Being on Earth:

- E.g. Mountain



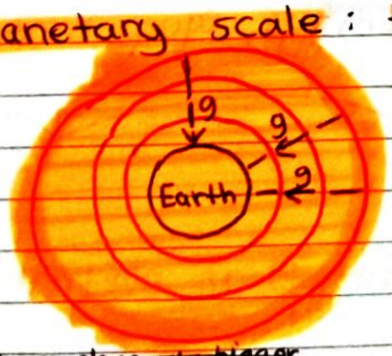
This is how equipotential surfaces are represented in 2D.



Equipotential surfaces of the mountain

Stacking the surfaces, rotating them, and tilting them, will produce a 3D image of the mountain.

- On planetary scale: the equipotential surfaces are spherical, not flat.



g is always perpendicular to every point on the equipotential surface V_g. (Good way to remember!)

~~Distance between plane gets bigger.~~

Hopefully, you have a good understanding of equipotential surfaces for gravitational field now.

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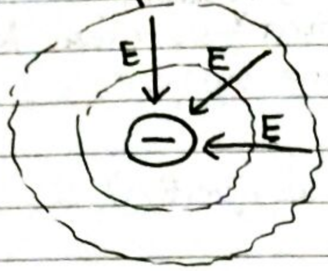
5). Equipotential surfaces - the electrical field

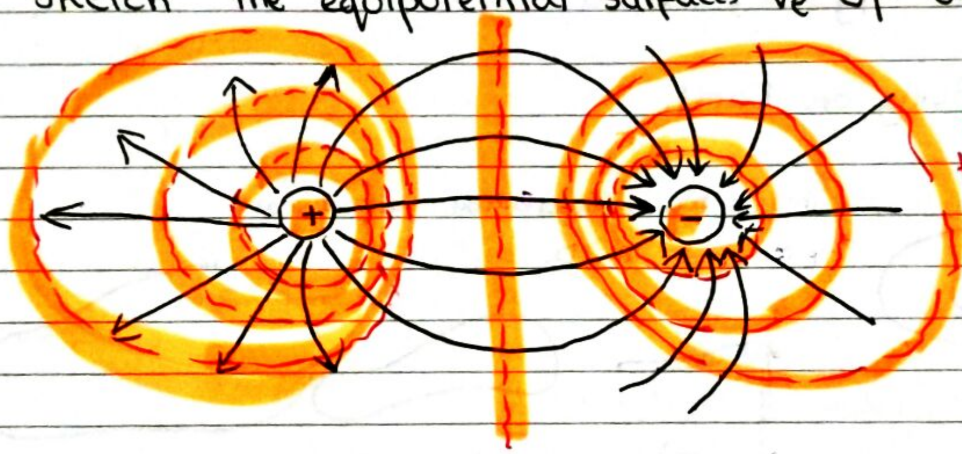
The surfaces are spheres just like with a planet.



The electric field vector **E** is perpendicular to every point on equipotential surface V_e.

Very important diagram next!

- Sketch the equipotential surfaces V_e of a dipole.

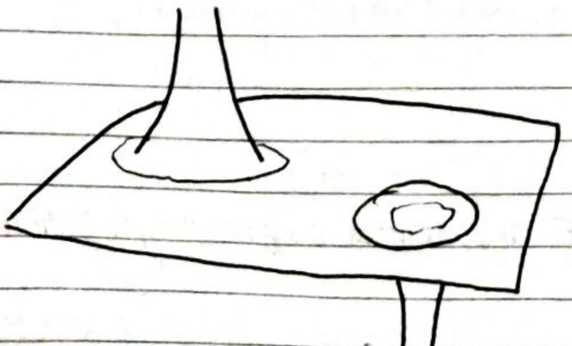


Just draw the surfaces perpendicular to the field lines

1) Learn it well. The surfaces are further apart on the other sides. ~~Not~~ In middle, they are close.
2) There is a surface in the very middle in form of a straight line. ~~perpendicular~~

Since E-field lines never cross or overlap, equipotential surfaces don't either.

Identify this equipotential surface.



This is an electric dipole.

The peak is the positive charge.

The pit is the negative charge.

For 2 masses, 2 − charges, or 2 + charges, equipotential surfaces are like shown in B. below. For a + & − charges, C is correct.

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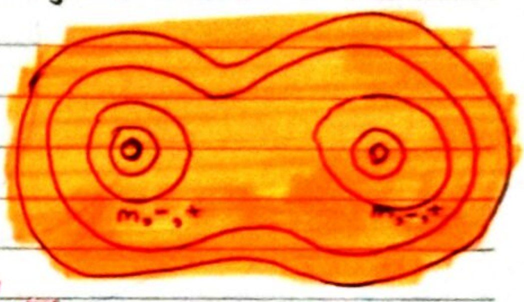
Which of the following diagrams best represents the equipotential surfaces due to two equal spherical charges?

A.

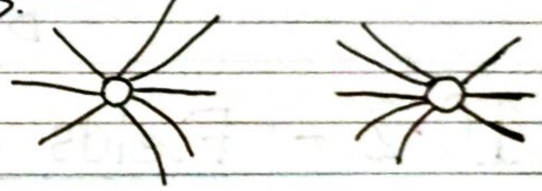
B.



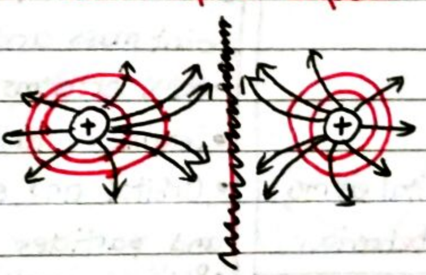
For two masses, −&− or +&+ charges. ~~Attractive.~~ The equip. surfaces are like that. Both masses attract.

C.

D.



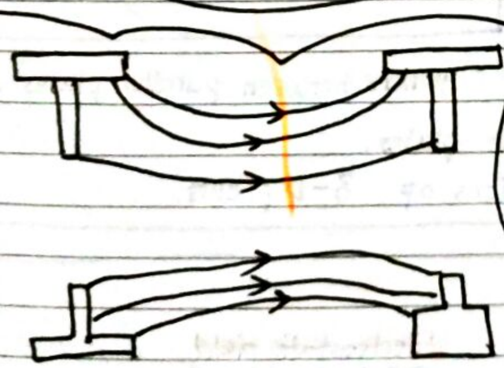
Firstly, since the charges are equal, this is not a dipole. C would be correct in case of a dipole. Let's try drawing the E-fields.

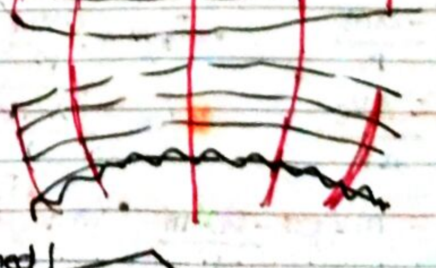


Since at the center point, there are no E-field lines, a vertical line is absent. B seems correct.

Answer. B

The diagram shows produced E-fields lines by an electrostatic focussing device. Draw equipotential lines ~~below on the~~ below.





Too far E-field to continue the equipotential surface at the edge. Hence, red line is not joined!

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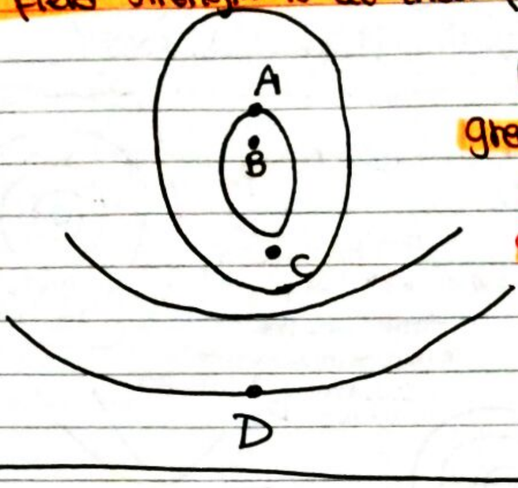
- Remember that the closer the equipotential surfaces, the strong the electric field strength is at that point.



Where is the E-field strength greatest?

At C, because the surfaces are closest there. Try and see the Image.

10.2 – Fields at work (Very large and important)

(Lots of stuff. Sometimes IB does that. Deal with it.)

(Extension of 5.1, ~~6.1~~, 6.2)

Understandings:

1) Potential and potential energy
2) Potential gradient
3) Potential difference
4) Escape speed
5) Orbital motion, orbital speed & orbital energy
6) Forces and inverse-square law behaviour

Applications and skills:

- Determine potential energy of a point mass and point charge.
- Solve problems with potential energy
- Determine potential inside a charged sphere.
- Orbital and escape speed – planetary and particles
- Problems involving forces on charges and masses in radial & uniform fields

Guidance:

1. We only look at circular orbit of a satellite.
2. ~~Know~~ Consider both uniform & radial fields.
3. Students should assume that E-field is uniform everywhere between parallel plates with edge effects occurring beyond the limits of the plates.
4. Lines of force can be 2-D ~~lines~~ representations of 3-D fields.

Data Booklet reference

Gravitational field	Electrostatic field
$V_g = \frac{-GM}{r}$	$V_e = \frac{kq}{r}$
$F_G = \frac{GMm}{r^2}$	$F_E = \frac{kq_1q_2}{r^2}$
$g = \frac{-\Delta V_g}{\Delta r}$	$E = \frac{-\Delta V_e}{\Delta r}$
$V_{esc} = \sqrt{\frac{2GM}{r}}$	$E_p = qV_e = \frac{kq_1q_2}{r}$
$E_p = mV_g = \frac{-GMm}{r}$	
$V_{orbit} = \sqrt{\frac{GM}{r}}$	

There are many formulas in this unit that are not in the data booklet. Memorize and understand all of them. Re-do the examples to better understand them.

Date

1. Potential energy - gravitational

- Potential energy is :

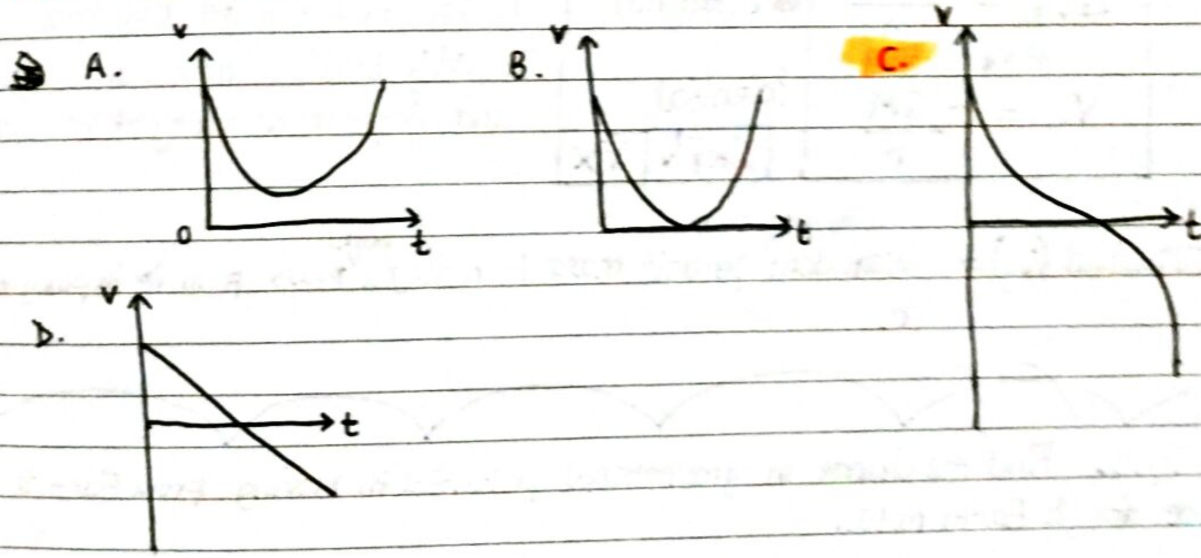
$E_p = -\frac{GMm}{r}$	Gravitational Potential Energy	Formula for large-scale gravitational fields. Like planet to planet.

Note, in particular, the minus sign.
That means $E_p = \max$ when $r = \infty$. That is $E_p = 0$.

Example

A spaceship moves directly away from a planet as shown below.

At point P, motors are turned off but spaceship remains under influence of the planet. Which graph represents best, the velocity v with time t of the spaceship after point P?



Solution

$F_g = \frac{GMm}{r^2}$

$F = ma_g$, $a_g = \frac{GM}{r^2}$

a is connected to v as it is slope of graph.

- The ship must slow down and reverse. So, v becomes −. A and B are wrong.
- Since force varies to $\frac{1}{r^2}$, a is not linear. D is wrong.

∴ Answer is C.

All potentials and energy are scalar. So, you add it up in vector questions without considering directions. But, forces are vectors. Fields are vectors. Potential gradient & field strength are also vectors.

KKY

Date

- "Local" or small scale formula for gravitational potential energy:

$\Delta E_p = mg\Delta y$	where $g = 9.81 ms^{-2}$	Local E_p

This formula only works when $g = $ CONSTANT, which is only true when Δy is small. From sea level to Mt. Everest, we consider Δy small.

For larger distances, use

ΔE_p for large distance where g is not constant	$\Delta E_p = -GMm\left(\frac{1}{r_f} - \frac{1}{r_0}\right) = -\frac{GMm}{r_f} + \frac{GMm}{r_0}$

2. Potential - gravitational

$E_p = -\frac{GMm}{r}$. Gravitational potential = Gravitational potential energy per unit mass:

$\Delta V_g = \frac{\Delta E_p}{m}$ kinda in 10.1 $W = m\,\Delta V_g$ $V_g = -\frac{GM}{r}$	Gravitational Potential Jkg^{-1} ✓ V ✗

Note the E_p is negative. So, ΔV_g is also negative. and V_g is also negative.

- G. Potential (V_g) = Work done by grav. field per unit mass in moving a small mass from infinity to r.

Example. Find the change in gravitational potential in moving from Earth's surface to 5 Earth radii.

$$\Delta V_g = -\frac{GM}{r_f} + \frac{GM}{r_0}$$

or

$$\frac{-\frac{GMm}{r_f} + \frac{GMm}{r_0}}{m} = -\frac{GM}{r_f} + \frac{GM}{r_0}$$

Same derivation in the end.

$$= -\frac{GM}{5r} + \frac{GM}{r}$$

$$\Delta V_g = \frac{4GM}{5r}$$

$M = 5.98\times10^{24}$ kg, $G = 6.67\times10^{-11}$, $r = 6.37\times10^{6}$

$$\Delta V_g = \frac{4(6.67\times10^{-11})(5.98\times10^{24})}{5(6.37\times10^{6})} = +5.01\times10^{7}\,Jkg^{-1}$$

$$-\frac{GM}{5r} + \frac{GM}{r}$$

$$\frac{4GM}{5r} = \frac{4\times6.67\times10^{-11}\times5.98\times10^{24}}{5\times(6.37\times10^{6})} = 5.01\times10^{7}\,Jkg^{-1}$$

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Basically, to calculate ΔE_p and ΔV_g, remember that

1) There are negative signs to be careful about.
2) It is final minus initial value. ~~So final - in~~ (Well the negative signs make it negative final plus initial value, but we need to understand the concept.)

In a nutshell, as you go far from earth, E_p and V_g are increasing since there is a negative sign. So when you go farther away, ΔE_p or ΔV_g is always positive (in gravitational fields).

3. Potential and Potential energy - gravitational

Which statement correctly defines gravitational potential at a point P in a gravitational field?

A. Work done per unit mass in moving a small mass from point P to infinity. by grav. field

B. Work done by grav. field per unit mass in moving a small mass from infinity to point P. gravitational potential

C. Work done by grav. field in moving a small mass from infinity to point P. Gravitational Potential Energy

D. Work done in moving a small mass from point P to infinity.

Gravitational potential is work done per unit mass (1), so C and D is wrong. Additionally, it is ~~from~~ for moving a mass from ∞ to a point P (1). So A is wrong.

Answer : B

(2 mark definition)

You must know this definition.

Btw, C is the perfect definition of gravitational potential energy at point P.

(No Calculator) The gravitational potential at point X is $-7\,kJ\,kg^{-1}$ and at point Y is $-3\,kJ\,kg^{-1}$. What is the change in gravitational potential energy if 4 kg mass is moved from X to Y?

$$\Delta V_g = \frac{\Delta E_p}{M}, \quad \Delta E_p = \Delta V_g \times m$$

$\Delta E_p = [-3+7][4] = 16\,kJ$

$$\Delta E_p = [-3-(-7)] \times 4 = 4 \times 4 = \boxed{16\,kJ}$$

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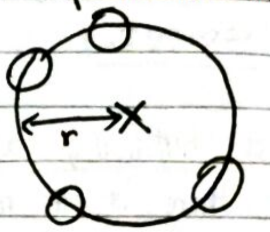
- Gravitational potential is a scalar. DON'T WORRY ABOUT DIRECTION.
  Gravitational potential energy is a scalar too. (Still ~~worry~~ consider + − though!)

(IMPORTANT)

Find the gravitational potential at the midpoint of the 2750 m radius circle of 125-kg masses shown.



$r = 2750\,m$

$M = 125\,kg$ ~~per m~~

Since V_g is a scalar, we don't cancel out. ~~due to direction like in electrostatic questions~~ There is no direction.

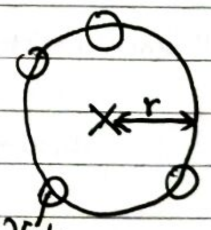
$$V_g = 4 \times \left(-\frac{GM}{r}\right) = 4 \times \left(\frac{-6.67\times10^{-11}\times125}{2750}\right)$$

$$= -4 \times 3.03\times10^{-12}$$

$$= -1.21\times10^{-11}\ J\,kg^{-1}$$

Don't forget the minus sign!

If a 365 kg mass is brought from infinity ∞ to the center of the circle of masses, how much potential energy would it have lost? $r = 2750\,m$.



125 kg

$$\Delta E_p = \left[\frac{GMm}{r_F} - \left(-\frac{GMm}{r_0}\right)\right] \times 4 \text{ (masses)}$$

$$\Delta E_p = \left[-\frac{GMm}{r_F} + \frac{GMm}{r_0}\right] \times 4$$

Path doesn't matter.
Mass could come from anywhere.

$$\Delta E_p = \left[-\frac{GMm}{2750} + \frac{GMm}{r_0}\right] \times 4$$

Remember the minus sign!
Just × 365 kg in previous answer.

$$= \left[-\frac{GMm}{2750} + 0\right] \times 4$$

$$= \frac{-6.67\times10^{-11}\times125\times365}{2750} \times 4$$

$$\Delta E_p = -4.42\times10^{-9}\ J$$

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4. Potential gradient - gravitational

Gravitational potential gradient (GPG) = the change in gravitational potential per unit distance.

DO NOT Memorize this. Go bottom of page later.

$$\boxed{g = \text{Gravitational } \cancel{\text{pote}} \text{ potential gradient} = \frac{\Delta V_g}{\Delta r} = \frac{\Delta \text{ Gravitational potential}}{\Delta \text{ distance}}}$$

$(J\,kg^{-1}\,m^{-1})$ or (ms^{-2})

~~Same~~

~~Not same~~

(See example below)

~~Basically, same formula as the gravitational field strength.~~ ~~But different definitions!~~

Example.

Q) Find GPG in moving the Earth's surface to 5 radii from Earth's ~~center~~ surface.

$M = 5.98\times10^{24}\ kg \qquad r = 6.37\times10^{6}\ m$

Solution

$\cancel{GPG = \frac{GM}{r^2} = \frac{6.67\times10^{-11}\times5.98\times10^{24}}{(6.37\times10^{6})^2}}$

$\cancel{= 9.83\ J\,kg^{-1}\,m^{-1}}$

$g,\ GPG = -\frac{\Delta V_g}{\Delta r}$

$$= \frac{-\frac{GM}{r_F} + \frac{GM}{r_0}}{5(6.37\times10^6) - 6.37\times10^6}$$

$$= \frac{5.01\times10^7}{4(6.37\times10^6)}$$

$$g = 1.97\ J\,kg^{-1}\,m^{-1}$$

Now if we were silly and said that it is same formula as grav. field strength, we have $GPG = \frac{GM}{r^2} = \frac{6.67\times10^{-11}\times5.98\times10^{24}}{25\times(6.37\times10^6)^2}$

$= 0.393$

Which is wrong!

However, the units for gravitational potential gradient are

$\boxed{ms^{-2}}$. Same units as ~~electric~~ gravitational field strength. Same symbol (g)

$J\,kg^{-1}\,m^{-1} = Nm\,kg^{-1}\,m^{-1} = N\,kg^{-1} = kg\,ms^{-2}\,kg^{-1} = ms^{-2}$

For planetary scale

$g = \frac{-\Delta V_g}{\Delta r}$	Gravitational Potential gradient ms^{-2}

See the minus sign.

That's the difference from the formula at top. This one is for planets. So −.

KIKY

Date

Example: The gravitational potential in vicinity of a planet changes from $-6.16\times10^{7}\ \text{Jkg}^{-1}$ to $-6.12\times10^{7}\ \text{Jkg}^{-1}$ in moving from 1.80×10^{8} m to 2.85×10^{8} m. What is the gravitational field strength in that region?

$$E = -\frac{\Delta V_g}{\Delta r}$$

$$= -\frac{\left[(-6.12\times10^{7})+(6.16\times10^{7})\right]}{(2.85\times10^{8})-(1.80\times10^{8})}$$

$$= -\frac{400000}{105000000}$$

$$E = -0.00381\ \text{Jkg}^{-1}\text{m}^{-1}$$

$$= -3.81\times10^{-3}\ \text{Jkg}^{-1}\text{m}^{-1} \text{ or } \text{ms}^{-2}.$$

Near Earth, $g = -\dfrac{\Delta V_g}{\Delta y}$

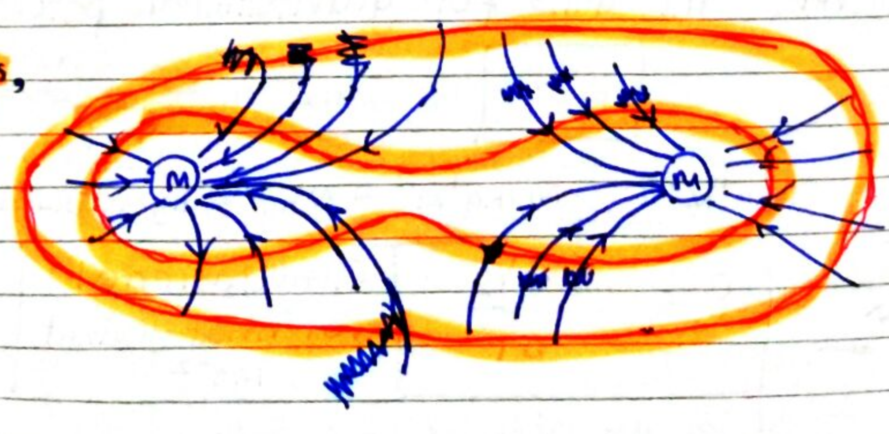
Planetary scale, $g = \dfrac{-\Delta V_g}{\Delta r}$

Gravitation potential gradient & electric field strength

5. Equipotential surfaces and potential gradient - gravitational

Remember that the equipotential surface rings get farther apart as we get farther from mass? You must!!!

For 2 masses,
- Field lines inwards.
- Equipotential surfaces are like in two + or two - charges.



Date

6. Escape Speed

- **Escape speed** – the minimum speed an object needs to escape a planet's gravitational pull.

  OR

  The minimum speed which will carry an object to infinity and bring it to rest there. – Perhaps good to remember for a multiple choice question.

$$\boxed{v_{esc} = \sqrt{\frac{2GM}{R}}} \quad \boxed{\text{Escape speed}}$$

Practice

Find the escape speed from Earth. $M = 5.98 \times 10^{24}$ kg, $R = 6.37 \times 10^{6}$ m.

$$v_{esc} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.37 \times 10^{6}}}$$

$$= 11190.7 \text{ ms}^{-1}$$

$$= 11200 \text{ ms}^{-1} \quad \cancel{= 11.2112 \text{ kms}^{-1}}$$

- Note : Escape speed is independent of the mass ~~it is actually escaping~~ ~~than~~ of the escaping object.

7. Orbital Motion, orbital speed and orbital energy

Gravitational force is the centripetal force for circular orbital motion.

From 6.1, we know that $\boxed{a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}} \quad \boxed{\text{Centripetal Acceleration}}$

Example. A 0.500 kg baseball is placed in a circular orbit around Earth at 8850 m above surface (Mount Everest). Given Earth has radius $R_E = 6400000$ m, find the speed of ball.

Solution. $F = \frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$, $v^2 = \frac{GM}{r}$, $v = \sqrt{\frac{GM}{r}}$

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6408850}} = 7890 \approx 7900 \text{ ms}^{-1}$$

KIKY WRONG. This is "Local" not planetary bro. T.O.

Date

F_c is caused by weight of ball (stationary g)

$F_c = 0.500 \times 9.81 = 4.905\ N$

$F_c = \frac{Mv^2}{r}$

$4.905 = \frac{Mv^2}{r}$

$9.81\ \cancel{0.500} = \frac{v^2}{r}$

$\sqrt{\frac{\cancel{0.500}}{9.81} \times 6408850} = v \approx 7930\ ms^{-1}$

When g is constant, we don't use planetary Formulas.

Again, don't use planetary formulas when g is constant.

g is constant for heights until Mt. Everest 8850m + 6400000m = 6408850m, g CONST

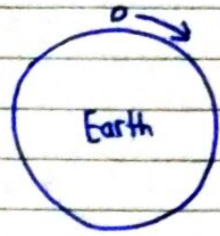
(KIKY)

Date

Practice.

Find the ~~orbit~~ period T of one complete orbit of a ~~the~~ ball that it is just above MT. EVEREST. R= 6400000M. Height of Everest = 8850M

$F_c = Mg$ or Ma_c (Not planetary)



$F_c = M \times 9.81 = 9.81M$

$F_c = \frac{Mv^2}{r}$, since $a_c = \frac{v^2}{r}$.

$9.81M = \frac{Mv^2}{r}$

$9.81r = v^2$

$v = \sqrt{9.81 \times 6408850}$

$= 7930 \, ms^{-1}$

$T = \frac{2\pi \times 6408850}{7930} = 5078$ seconds

$T \approx 5080$ seconds or 1.4 hours

Prove Kepler's Third Law (For an object in circular orbit)

•1 In circular orbit $F_c = Ma_c$ and $F_c = \frac{GMm}{r^2}$

•2 $a_c = \frac{4\pi^2 r}{T^2}$. Then,

IBO Expects you to derive this!

•3 $\cancel{m}a_c = \frac{GM\cancel{m}}{r^2}$

$a_c = \frac{GM}{r^2}$

To derive Kepler's Law, you ~~derive~~ use the planetary and local F_c formulas. $ma_c = \frac{GMm}{r^2}$

•4 $\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}$

$a_c = \frac{GM}{r^2}$

$a_c = \frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}$

•5 $4\pi^2 r^3 = GMT^2$

$\boxed{T^2 = \left[\frac{4\pi^2}{GM}\right] r^3}$ Kepler's Third Law

Date

Practice: Using Kepler's third law, find the period T of one complete orbit of the baseball from the previous example.

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$r = 6408850 m$ $M = 5.98 \times 10^{24} kg$

$$T = \sqrt{\frac{4\pi^2}{GM} \times 6408850^3}$$

$T = 5100$ seconds (Slight ~~discrepancy~~ discrepancy)

So, when exist questions regarding T and r, period and radius, use Kepler's Third Law.

Right! ~~Net~~ Next concept

Orbital energy

Gravitational potential energy $= E_p = -\frac{GMm}{r}$, where M is Mass of Earth and m is mass of object.

~~Gravit~~ Kinetic energy of an object of mass m moving at speed v is

$$E_K = \frac{1}{2}mv^2$$

Thus, the mechanical energy of an orbiting satellite of mass m is:

$E = E_K + E_p$ $E = \frac{1}{2}mv^2 - \frac{GMm}{r}$	Total Energy of an orbiting satellite (Not in D.B.)(Remember it!)

Date

Example: Show that the speed of an orbiting satellite having mass m at a distance r from the center of Earth (mass M) is $V_{orbit} = \sqrt{\frac{GM}{r}}$

~~$E = E_K + E_p$~~

~~$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$~~

$$F_c = \frac{mv^2}{r} = ma_c$$

$$F_G = +\frac{GMm}{r^2}$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

$V_{orbit} = \sqrt{\frac{GM}{r}}$	Speed of an orbiting satellite	~~Not in the D.B (Remember)~~

In circular orbit $F_c = ma_c$ and $F_c = \frac{GMm}{r^2}$

But $a_c = \frac{v^2}{r}$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$mv^2 = \frac{GMm}{r}$$

$$KE = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$E_K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$	Kinetic energy of an orbiting satellite	Not in D.B. Remember it!

$$E = E_K + E_p = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$E = -\frac{GMm}{2r}$$

~~Remeber~~ Remember it!

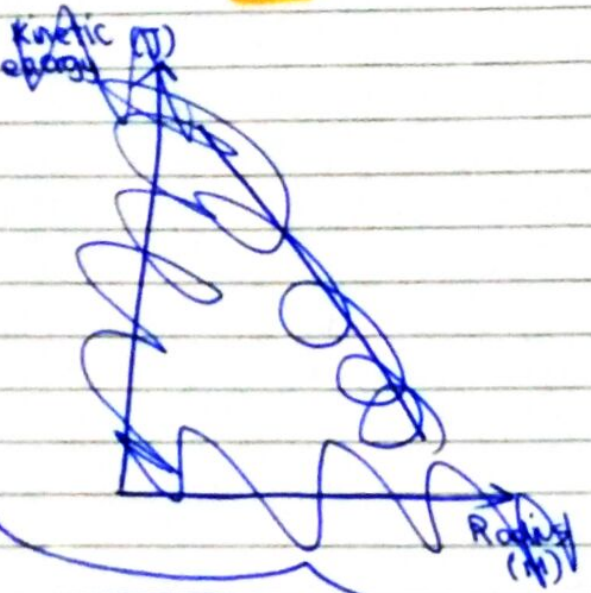
$E = \frac{-GMm}{2r}$ $E_K = \frac{GMm}{2r}$ $\quad E_p = \frac{-GMm}{r}$	Total energy of an orbiting satellite	Not in D.B

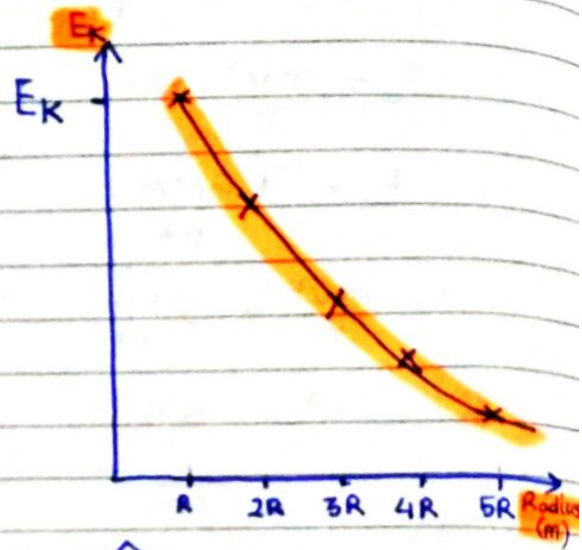
Example

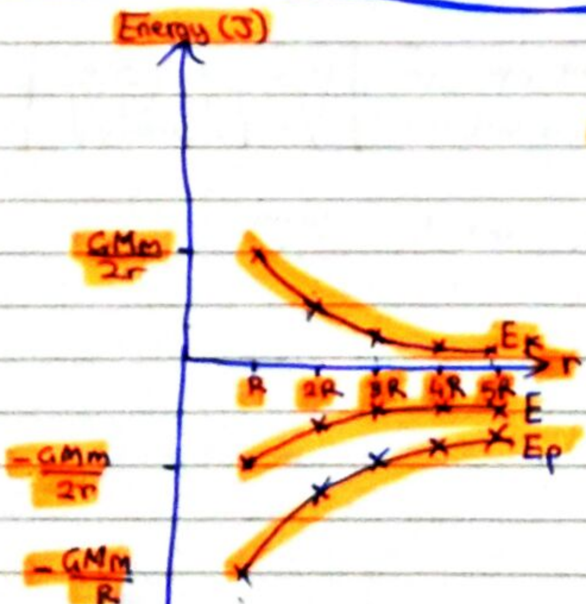
Graph the kinetic energy vs radius of orbit for a satellite of mass m about a planet of mass M and radius R.

So, its like solar system. Mercury is fast & Neptune is slow.

$E_K = \frac{GMm}{2r}$ $\qquad E_K \propto \frac{1}{2r}$







E_K decreases with radius.

E increases with radius.

E_p increases with radius.

8) Orbital speed and Weightlessness

If dobson falls at $10\ ms^{-2}$ with a ball, he observes the ball's acceleration to be zero. This is the same as weightlessness in an orbiting spacecraft.

The astronaut, spacecraft & tomatoes, accelerate at $a_c = g$. They all fall together and appear to be weightless.

Date

Q. Which of the following relate the Radius r of the circular orbit of a planet round a star to the period T of the orbit?

$\boxed{A.}$ $R^3 \propto T^2$ B. $\frac{1}{R^3} \propto T^2$ C. $R^2 \propto T^3$ D. $\frac{1}{R^2} \propto T^3$

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$$= \underline{T^2 \propto r^3}$$

Q. Orbital ~~speed~~ period of Satellite X is 8 times that of Satellite Y.

The ratio $\frac{\text{orbital radius of satellite X}}{\text{orbital radius of satellite Y}}$ is

A. 2 $\boxed{B.}$ 4 C. 8 D. 16

$T \propto r^{1.5}$

$8T \propto r_2^{1.5}$

$8T \propto (4)^{1.5}$

$\cancel{v_{orb} = \sqrt{\frac{GM}{r}}}$

$\cancel{8v_{orb} = \sqrt{\frac{GM}{\frac{r}{64}}} = 8\sqrt{\frac{GM}{r}}}$

$\cancel{\frac{\frac{r}{64}}{r} = \frac{1}{64}}$

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$$(8T)^2 = \frac{4\pi^2}{GM}(4r)^3$$

$$\frac{4}{1} = 4$$

A satellite of mass m and speed v orbits Earth at distance r from centre of Earth. Gravitational potential strength due to the Earth at the satellite is equal to

A. $\frac{v}{r}$ $\boxed{B.}$ $\frac{v^2}{r}$ C. $\frac{mv}{r}$ D. $\frac{mv^2}{r}$

$$v_{orb} = \sqrt{\frac{GM}{r}}$$

$$v^2 = \frac{GM}{r}$$

$$\boxed{\frac{v^2}{r}} = \frac{GM}{r^2}$$

Date

a) State the effect of frictional forces of atmosphere on an orbiting object.

Solution

The satellite begins losing mechanical energy as a form of heat.

b) Deduce that, as a result of this friction, the radius will change continuously.

$$E = \frac{-GMm}{2r}$$

As E decreases, radius decreases. (Due to negative sign.)

Radius will continuously decrease too. As it decreases, orbital velocity increases & atmosphere gets thicker & more resistive. Clearly, the orbit begins to decay (shrink).

Q. A satellite it in orbit about Earth. What happens to KE and PE when satellite moves an orbit closer to Earth?

	ΔPE	ΔKE
A	↓	↑
B	↓	↓
C	↑	↑
D	↑	↓

Sol.

$KE = \frac{GMm}{2r}$, reducing r increases KE.

$PE = -\frac{GMm}{r}$, reducing r decreases PE. Negative sign.

Ans : A

Q. Space probe is launched from equator in direction of North pole of Earth. Energy E given to the space probe of mass m is $E = \frac{3GMm}{4R_e}$, G is gravitational constant, M is mass of Earth, R_e is radius of Earth. Work done in overcoming frictional forces is not considered.

a) i) What is meant by escape speed.

The minimum speed required to bring an object from a surface to infinity & bring it to rest there. OR

Minimum speed needed to escape the gravitational pull of a planet.

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

Date

ii) Deduce that the space probe won't be able to travel into deep space.

The escape speed $v = \sqrt{\frac{2GM}{R_e}}$

Needed $KE = \frac{1}{2}mv_{esc}^2 \cancel{= \frac{GMm}{r} = \frac{GMm}{R_e}}$

Needed $KE = \frac{1}{2}m\left(\sqrt{\frac{2GM}{R_e}}\right)^2 = \frac{GMm}{R_e}$

This is the KE alone and the provided energy is $\frac{3}{4}$ of the needed KE only. Hence, it won't travel into deep space.

b) The space probe is launched into a circular polar orbit of radius R.
Derive expressions, in G, M, R_e, m & R, for:

i) ΔGPE in travelling from Earth's surface to its orbit.

$GPE_i = -\frac{GMm}{R_e}$, $GPE_f = -\frac{GMm}{R}$

$\Delta GPE = -\frac{GMm}{R} + \frac{GMm}{R_e} = GMm\left(\frac{1}{R_e} - \frac{1}{R}\right)$

ii) KE of probe in orbit.

$KE_i = \frac{GMm}{2R_E}$ $KE_f = \frac{GMm}{2R}$

$\boxed{KE_f = \frac{GMm}{2R}}$

c) Using your answers & question, determine height of orbit above the Earth's surface.

$V_{orb} = \sqrt{\frac{GM}{R}}$ Used $KE = \frac{1}{2}m\left(\frac{GM}{R}\right) = \frac{GMm}{2R}$, Used $PE = GMm\left(\frac{1}{R_e} - \frac{1}{R}\right)$

$KE + PE = E$

$\frac{3GMm}{4R_e} = \frac{GMm}{2R} + GMm\left(\frac{1}{R_e} - \frac{1}{R}\right)$

The height above the Earth's surface is R_e.

$\frac{3}{4R_e} = \frac{1}{2R} + \frac{1}{R_e} - \frac{1}{R}$

$\frac{1}{2R} = \frac{1}{4R_e}$, $R = 2R_e$

KIKY

Date

Q a) Define gravitational potential.

Work done by gravitational field per unit mass in moving a point mass from infinity to that point. ~~one point to another~~

From one point to another point is g.potential difference.

COMPARE : GPE : Work done by gravitational field in moving a small mass from infinity to that point.

The definitions only differ by "per unit mass".

b) Planet has mass M and radius R_0. Magnitude g_0 of gravitational field strength at surface of a planet is

$$g_0 = \frac{GM}{R_0^2}$$

Show that gravitational potential V_0 at surface is $V_0 = -g_0 R_0$

$$V_0 = -\frac{GM}{R_0}$$

$$g_0 = \frac{GM}{R_0^2}$$

$$V_0 = \frac{GM}{R_0^2}(-R_0) = -\frac{GM}{R_0}$$

$$V_0 = -g_0 R_0$$

c) In a graph, using largest values or the most ~~same~~ appropriate one. Remember this!

$$V_0 = -g_0 R_0$$

If there is V_0 by g_0 graph,
use largest values or use V_0 at R_0, V_0,
to determine g_0, which is $-\frac{V_0}{R_0}$.

KIKY

Date

8) Potential and Potential Energy - electrostatic

$E_p = \dfrac{kq_1q_2}{r}$ where $k = 8.99 \times 10^9\ Nm^2C^{-2}$	Electrostatic Potential Energy

No need to prove it.

Find the electric potential energy between 2 protons located 3.0×10^{-10} m apart.

$$E_p = \frac{kq_1q_2}{r} = \frac{(8.99 \times 10^9)(1.6 \times 10^{-19})^2}{3.0 \times 10^{-10}}$$

$$E_p = 7.67 \times 10^{-19}\ J \approx 7.7 \times 10^{-19}\ J$$

- Electrostatic potential energy - work done by electrostatic field in bringing a point ~~mass~~ charge from infinity to that point.
- Electrostatic potential V_e - work done per unit ~~mass~~ charge by electrostatic field in moving a point ~~mass~~ charge from infinity to that point. Scalar! (Though can be negative)

$\Delta V_e = \dfrac{\Delta E_p}{q}$ $V_e = \dfrac{kq}{r}$	Electrostatic Potential

Since point charge is usually brought from infinity, we consider $\Delta E_p = E_p$ sometimes.

Q a) Find electrical potential at point P located 4.5×10^{-10} m from a proton.

$$V_e = \frac{8.99 \times 10^9 \times 1.6 \times 10^{-19}}{4.5 \times 10^{-10}} = 3.20 \text{ Volts or } JC^{-1}$$

b) Find electrical potential energy when there is an electron is at point P.

$$V_e = \frac{\Delta E_p}{q}$$

$$\Delta E_p = 3.20 \times (-1.6 \times 10^{-19})$$

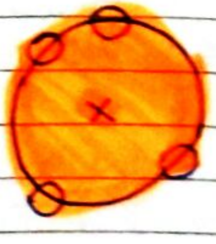
$$E_p = -5.11 \times 10^{-19}\ J \approx -5.1 \times 10^{-19}\ J$$

Date

- ~~Sine~~ Since V_e is a scalar, finding electric potential with more than one point charge is simply an additive process.

Example

Q) Find electric potential at center of circle of protons. Radius of circle is size of a small nucleus, or 3.0×10^{-15} m.

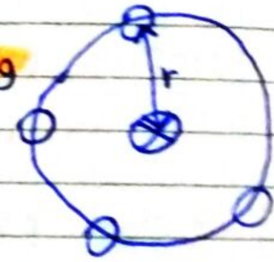


$$V_e = \frac{kq}{r} \times 4$$

$$V_e = \frac{8.99 \times 10^{9} \times 1.6 \times 10^{-19}}{3.0 \times 10^{-15}} \times 4$$

$$V_e = 1.9 \times 10^{6}\ J\,C^{-1} \text{ or } V$$

Q) Find change in electrical potential energy (MeV) in moving a proton from infinity to center of previous nucleus.



~~ΔV_e~~

~~$\Delta PE = V_e \times q$~~

~~$\Delta PE = \frac{(1.9 \times 10^{6})(9.32)}{1.673 \times 10^{-27}}$~~

~~$PE = \Delta PE = \frac{1.9 \times 10^{6}}{1.661 \times 10^{-27}} \times 931.5 \times 1.6 \times 10^{-19}$~~

~~$\Delta PE = 1.7 \times 10^{17}$ MeV~~

$$\Delta PE = V_e \times q$$

~~1 J~~ $1\ V = 1\ J\,C^{-1}$

1.6×10^{-19} J $= 1$ eV

$$\Delta PE = \frac{V_e \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$\Delta PE = 1.9 \times 10^{6}\ eV = 1.9\ MeV$$

~~Since it is moved from ∞ to r, ΔPE = PE~~

Since it is moved from ∞ to r, ΔPE = PE.

9) Potential Gradient - electrostatic

- Electric potential gradient - change in electric potential per unit distance.

Recall relationship between EPG and electric field strength.

$E = \frac{-\Delta V_e}{\Delta r}$	~~Electrostatic field strength~~	Electrostatic Potential Gradient

KIKY

Date

Q) The electric potential in vicinity of a charge changes from -3.75V to -3.63V in moving from $r = 1.80 \times 10^{-10}$ m to $r = 2.85 \times 10^{-10}$ m. What is the electric field strength in that region?

$$E = -\frac{\Delta V_e}{\Delta r} = \frac{-(-3.63 + 3.75)}{(2.85 \times 10^{-10}) - (1.80 \times 10^{-10})}$$

$$E = -1.14 \times 10^{9}\ Vm^{-1}$$

It can be negative.

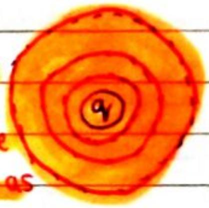
Did you know that the E-field is ZERO inside a conductor. Like ~~electron~~ a sphere, etc.

10) Equipotential Surfaces Visited - Electrostatic

Equipotential surfaces - imaginary surfaces ~~wh~~ with same electric potential.

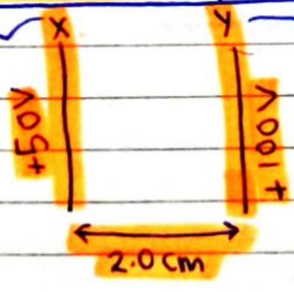
Since V_e is constant, r must be constant. Hence, the surfaces are concentric spheres.

The change in Ve of consecutive surfaces is equal. Since $V_e \propto \frac{1}{r}$, the conseutive rings get further apart as we get further from mass.



11) Equipotential Surfaces and the potential gradient

Q



not equi

Two electropotential surfaces. What is the ~~magnitude~~ & direction of electric field & its strength?

Sol.

Direction is + to -. $\boxed{Y \to X}$

$$E = -\frac{\Delta V_e}{\Delta x} = E = \frac{50}{2} = \boxed{25\ V\,cm^{-1}}$$

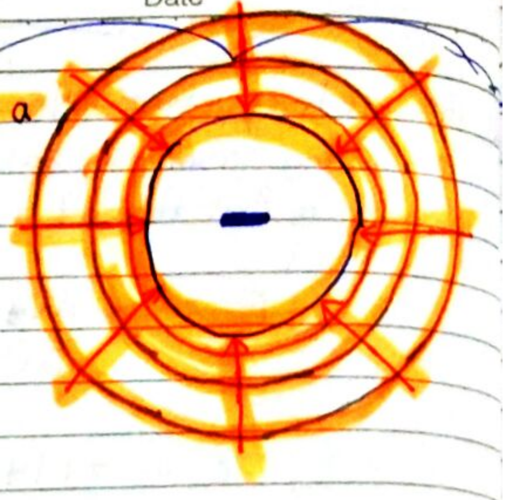
Define electric potential.

The work done per unit charge by electric field in bringing a point charge from infinity to that point. By contrast, electrical potential energy does not include "per unit charge".

KIKY

Date

Q) An isolated, metal sphere in a vacuum carries a negative electric charge of 9.0 nC.



b i) Draw arrows to represent electric field pattern in region outside the charged sphere.

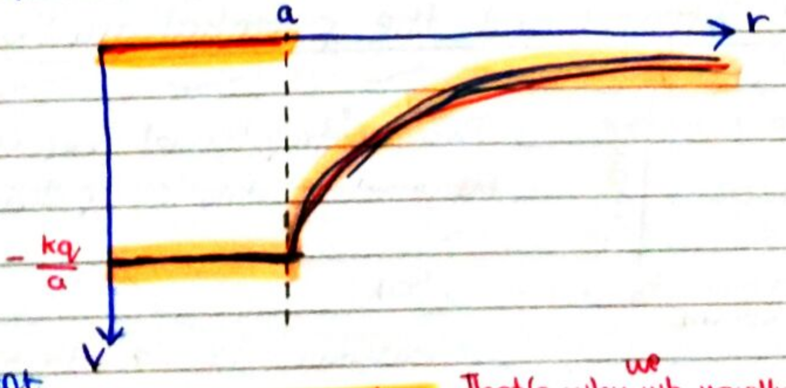
ii) Draw 3 equipotential surfaces outside sphere. The PDs between the lines must be equal. Sol. Perpendicular to E-field lines, and spreading.

c) Explain how the lines representing equipotential surfaces show that the strength of the electric field is decreasing with distance from centre of the sphere.

The electric potential is inversely proportional to distance r from the centre. So, We see that the bigger the separation between consecutive circles, the weaker the E-field and hence, the E-field decreases with distance from centre of sphere.

You can also tell directly from concentration of the E-field lines.

If a is the radius of a conductor, then the graph for electrical potential V_e is like this:



~~flat, for~~

- V_e is ~~zero~~ constant inside a conductor. That's why ~~we~~ we usually make it a point charge.
- V_e is ~~smallest (−)~~ smallest (−) when $r = a$. Inversely proportional to r.

Date

Q) Describe path of followed by an electron as it leaves surface of the sphere (from previous page). Charge of $-9.0\,nC$. Radius $= 4.5 \times 10^{-2}$ M

It accelerates away with a dropping inversely proportional to r^2, in a straight radial line.

Q) Determine the speed of the electron when it reaches a point at distance 0.30M from the centre of the sphere.

~~ΔEPE = ΔKE~~

$$\cancel{\Delta E = \frac{kQq}{Q} - kQq}$$

$$\cancel{\Delta E_p = \frac{kq_1 q_2}{r} = \frac{9.0\times10^9 \times (-9.0\times10^{-9}) \times (-1.6\times10^{-19})}{0.30}}$$

$\cancel{\Delta E_p = \Delta}$

$$\Delta E_p = \cancel{\Delta KE}\ \Delta E_K$$

$$\Delta E_p = \Delta V_e q$$

$$\Delta E_p = \left(\frac{kq_1}{0.3} - \frac{kq_1}{0.045}\right) q_2$$

$$\Delta E_p = \left(\frac{-81}{0.3} + \frac{81}{0.045}\right) q_2$$

$$\Delta E_p = 1530\, q_2$$

$$\Delta E_p = 1530 \times -1.6 \times 10^{-19}$$

$$\Delta E_p = -2.4 \times 10^{-16}\ J$$

$$\Delta E_K = 2.4 \times 10^{-16}\ J - 0 = 2.4 \times 10^{-16}\ J$$

$$2.4\times10^{-16} = \frac{1}{2}(9.11\times10^{-31})v^2$$

$$\frac{4.8\times10^{-16}}{9.11\times10^{-31}} = v^2$$

$$v = 2.3 \times 10^{7}\ ms^{-1}$$

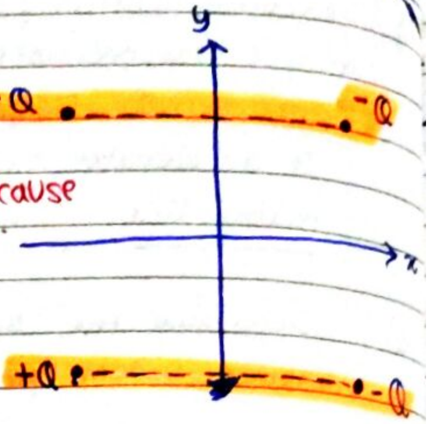
Knowledge!

Electrical potential gradient at a point is numerically equal to the electric field strength at that point.

KIKY

Date

Q) Four point charges are shown. At which position or positions is the electric potential zero?



This is 0 on y-axis because of + & −. Not direction.

Sol.

$V_e = +\frac{kq}{r}$

So, it is 0 on $\boxed{\text{y-axis}}$ since r is same and the paired Qs are opposite.